



**Coimisiún na Scrúduithe Stáit**  
**State Examinations Commission**

**Leaving Certificate 2013**

**Marking Scheme**

**Mathematics**  
**(Project Maths – Phase 3)**

**Higher Level**

### **Note to teachers and students on the use of published marking schemes**

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

### **Future Marking Schemes**

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

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Coimisiún na Scrúduithe Stáit  
State Examinations Commission

Leaving Certificate Examination 2013

Mathematics  
(Project Maths – Phase 3)

Paper 1

Higher Level

Friday 7 June      Afternoon 2:00 – 4:30

300 marks

## Model Solutions – Paper 1

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.

## Instructions

There are **two** sections in this examination paper:

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **all six** questions from this section.

**Question 1****(25 marks)**

$z = \frac{4}{1+\sqrt{3}i}$  is a complex number, where  $i^2 = -1$ .

(a) Verify that  $z$  can be written as  $1-\sqrt{3}i$ .

$$z = \frac{4}{1+\sqrt{3}i} = \frac{4}{1+\sqrt{3}i} \times \frac{1-\sqrt{3}i}{1-\sqrt{3}i} = \frac{4-4\sqrt{3}i}{1+3} = 1-\sqrt{3}i$$

**OR**

$$\text{If } z = \frac{4}{1+\sqrt{3}i} = 1-\sqrt{3}i$$

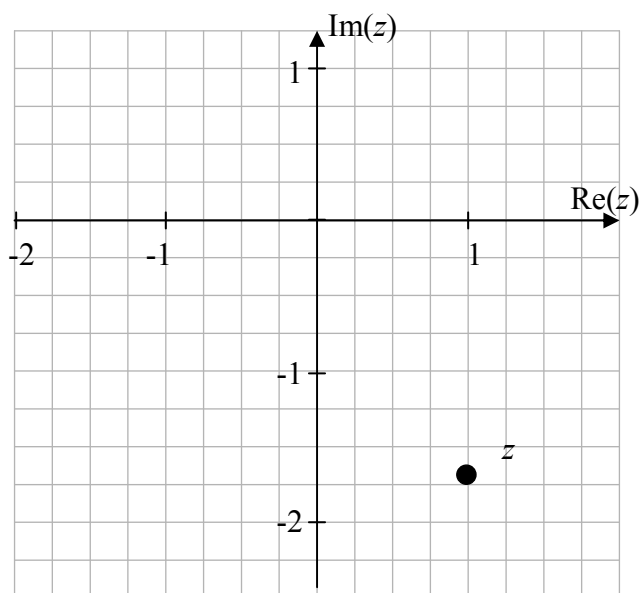
$$\text{then } 4 = (1+\sqrt{3}i)(1-\sqrt{3}i) = (1)^2 + (\sqrt{3})^2 = 4 \\ \Rightarrow \text{True}$$

(b) Plot  $z$  on an Argand diagram and write  $z$  in polar form.

$$\tan \alpha = \frac{\sqrt{3}}{1} \Rightarrow \alpha = \frac{\pi}{3} \Rightarrow \theta = \frac{5\pi}{3}$$

$$r = |1-\sqrt{3}i| = \sqrt{1+3} = \sqrt{4} = 2$$

$$z = 2 \left( \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right)$$



(c) Use De Moivre's theorem to show that  $z^{10} = -2^9(1 - \sqrt{3}i)$ .

$$\begin{aligned} z^{10} &= \left[ 2 \left( \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right) \right]^{10} \\ &= 2^{10} \left( \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right) = 2^{10} \left( \cos \frac{50\pi}{3} + i \sin \frac{50\pi}{3} \right) \\ &= 2^{10} \left( \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right) = 2^{10} \left( -\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) = -2^9(1 - \sqrt{3}i) \end{aligned}$$



**Question 2**

**(25 marks)**

(a) Find the set of all real values of  $x$  for which  $2x^2 + x - 15 \geq 0$ .

$$2x^2 + x - 15 = 0$$

$$\Rightarrow (2x - 5)(x + 3) = 0 \Rightarrow x = 2\frac{1}{2} \text{ or } x = -3$$

$$2x^2 + x - 15 \geq 0 \text{ for } \{x \mid x \leq -3\} \cup \{x \mid x \geq 2\frac{1}{2}\}$$

**OR**

$$f(x) = 2x^2 + x - 15 = (2x - 5)(x + 3)$$

$$(2x - 5)(x + 3) = 0$$

$$\Rightarrow x = \frac{5}{2} \text{ or } x = -3$$

(i):  $x \geq -3$  and  $x \geq \frac{5}{2} \Rightarrow x \geq \frac{5}{2}$

(ii):  $x \leq -3$  and  $x \leq \frac{5}{2} \Rightarrow x \leq -3$

Solution Set:  $\{x \mid x \leq -3\} \cup \{x \mid x \geq \frac{5}{2}\}$

(b) Solve the simultaneous equations,

$$\begin{aligned} x + y + z &= 16 \\ \frac{5}{2}x + y + 10z &= 40 \\ 2x + \frac{1}{2}y + 4z &= 21. \end{aligned}$$

$$\begin{array}{rcl} x + y + z = 16 & \Rightarrow & 2x + 2y + 2z = 32 \\ \frac{5}{2}x + y + 10z = 40 & \Rightarrow & 5x + 2y + 20z = 80 \\ & & \hline & & 3x \quad + 18z = 48 \end{array}$$

$$\begin{array}{rcl} x + y + z = 16 & & \\ 4x + y + 8z = 42 & & \\ \hline 3x \quad + 7z = 26 & & \end{array}$$

$$\begin{array}{rcl} 3x + 18z = 48 & & \\ 3x + 7z = 26 & & \\ \hline 11z = 22 \Rightarrow z = 2 & & \end{array}$$

$$3x + 7z = 26 \Rightarrow 3x + 7(2) = 26 \Rightarrow 3x = 12 \Rightarrow x = 4$$

$$x + y + z = 16 \Rightarrow 4 + y + 2 = 16 \Rightarrow y = 10$$

**Question 3****(25 marks)**

Scientists can estimate the age of certain ancient items by measuring the proportion of carbon-14, relative to the total carbon content. The formula used is  $Q = e^{-\frac{0.693t}{5730}}$ , where  $Q$  is the proportion of carbon-14 remaining and  $t$  is the age, in years, of the item.

- (a) An item is 2000 years old. Use the formula to find the proportion of carbon-14 in the item.

$$Q = e^{-\frac{0.693t}{5730}} = e^{-\frac{0.693 \times 2000}{5730}} = 0.7851$$

- (b) The proportion of carbon-14 in an item found at Lough Boora, County Offaly, was 0.3402. Estimate, correct to two significant figures, the age of the item.

$$\begin{aligned} Q &= e^{-\frac{0.693t}{5730}} = 0.3402 \\ \Rightarrow -\frac{0.693t}{5730} &= \ln 0.3402 \\ \Rightarrow t &= -\frac{5730 \times \ln 0.3402}{0.693} \approx 8915 \approx 8900 \text{ years} \end{aligned}$$

**Question 4****(25 marks)**

- (a) Niamh has saved to buy a car. She saved an equal amount at the beginning of each month in an account that earned an annual equivalent rate (AER) of 4%.
- (i) Show that the rate of interest, compounded monthly, which is equivalent to an AER of 4% is 0.327%, correct to 3 decimal places.

$$(1+i)^{12} = 1.04 \Rightarrow 1+i = \sqrt[12]{1.04} = 1.003273 \Rightarrow i = 0.003274$$

Hence,  $i = 0.327\%$

**OR**

$$\begin{aligned} (1.00327)^{12} &= 1.039953481 \\ &= 1.0400 \end{aligned}$$

$$r = 4\%$$

- (ii) Niamh has €15 000 in the account at the end of 36 months. How much has she saved each month, correct to the nearest euro?

$$\begin{aligned} 15000 &= P(1.00327^{36} + 1.00327^{35} + \dots + 1.00327^2 + 1.00327) \\ \Rightarrow P \left[ \frac{1.00327(1.00327^{36} - 1)}{1.00327 - 1} \right] &= 15000 \\ \Rightarrow P[38.26326387] &= 15000 \\ \Rightarrow P = 392.02 &= \text{€}392 \end{aligned}$$

**OR**

- Amortisation:

Step 1: Present Value

$$P = \frac{F}{(1+i)^t}$$

$$P = \frac{15000}{(1.04)^3} = 13334.95 \quad \text{OR} \quad P = \frac{15000}{(1.00327)^{36}} = 13336.73$$

Step 2:

$$A = \frac{(13334.95)(0.00327)(1.00327)^{36}}{1.00327^{36} - 1}$$

$$= €393.25$$

$$= €393$$

**OR**

- Present Value

$$P = \frac{F}{(1+i)^t}$$

$$P = \frac{15000}{(1.04)^3} = 13334.95$$

$$13334.95 = A \left( \frac{1}{1.00327} + \frac{1}{(1.00327)^2} + \dots + \frac{1}{(1.00327)^{36}} \right)$$

$$13334.95 = A \left[ \frac{\frac{1}{1.00327} \left( 1 - \left( \frac{1}{1.00327} \right)^{36} \right)}{1 - \frac{1}{1.00327}} \right]$$

$$A = €393.25$$

$$A = €393$$

- (b) Conall borrowed to buy a car. He borrowed €15 000 at a monthly interest rate of 0.866%. He made 36 equal monthly payments to repay the entire loan. How much, to the nearest euro, was each of his monthly payments?

$$\begin{aligned} A &= P \frac{i(1+i)^t}{(1+i)^t - 1} \\ &= 15000 \left[ \frac{0.00866(1+0.00866)^{36}}{1.00866^{36} - 1} \right] \\ &= 486.77 \end{aligned}$$

Monthly payment €487

**OR**

$$\begin{aligned} 15000 &= P \left( \frac{1}{1.00866} + \frac{1}{1.00866^2} + \dots + \frac{1}{1.00866^{36}} \right) \\ \Rightarrow P \left[ \frac{\frac{1}{1.00866} \left( 1 - \frac{1}{1.00866^{36}} \right)}{1 - \frac{1}{1.00866}} \right] &= 15000 \end{aligned}$$

$$\Rightarrow P[30.8151777] = 15000$$

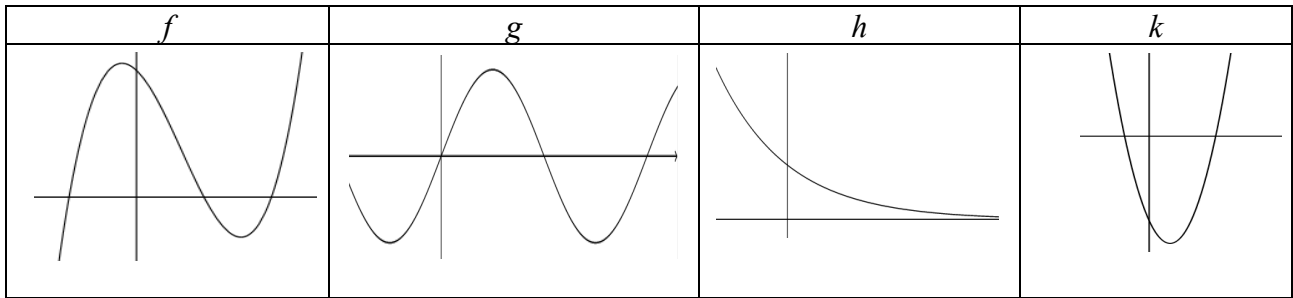
$$\Rightarrow P = 486.77$$

Monthly payment €487

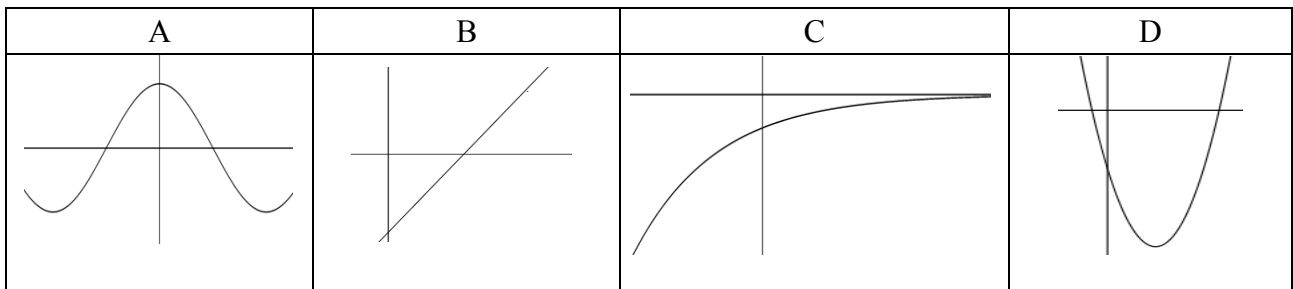
**Question 5**

**(25 marks)**

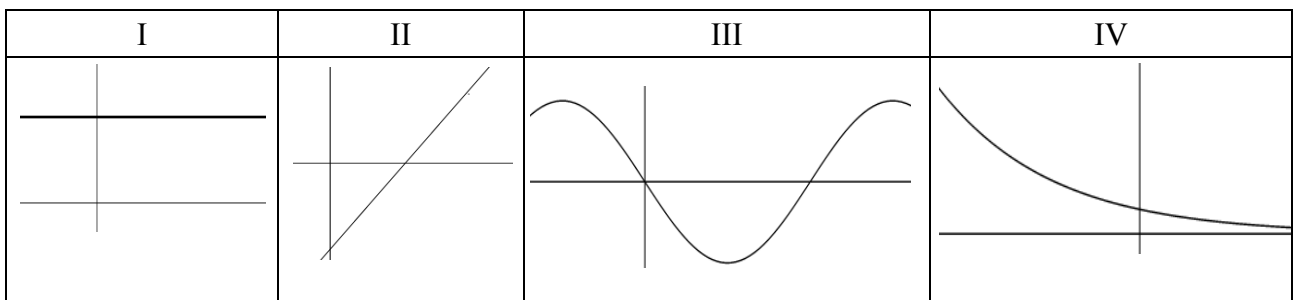
Each diagram below shows part of the graph of a function. Each is one of these: quadratic, cubic, trigonometric or exponential (not necessarily in that order).



Each diagram below shows part of the graph of the first derivative of one of the above functions (not necessarily in the same order).



Each diagram below shows part of the graph of the second derivative of one of the original functions (not necessarily in the same order).



**(a)** Complete the table below by matching the function to its first derivative and its second derivative.

Type of function	Function	First derivative	Second derivative
Quadratic	<i>k</i>	B	I
Cubic	<i>f</i>	D	II
Trigonometric	<i>g</i>	A	III
Exponential	<i>h</i>	C	IV

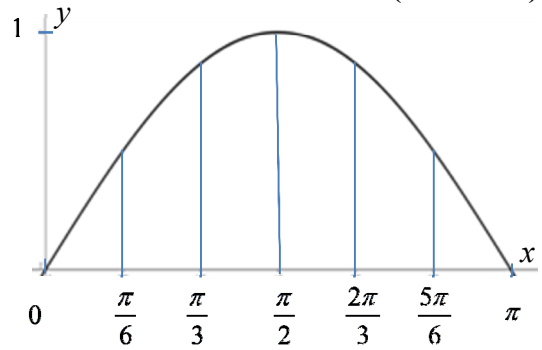
**(b)** For **one** row in the table, explain your choice of first derivative and second derivative.

A quadratic function differentiates to a line which differentiates to a constant.

**Question 6**

**(25 marks)**

The diagram shows the graph of the function  $y = \sin x$  in the domain  $0 \leq x \leq \pi$ ,  $x \in \mathbb{R}$ .



- (a) Complete the table below, correct to three decimal places.

$x$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	$\pi$
$y$	0	0.5	0.866	1	0.866	0.5	0

- (b) Use the trapezoidal rule to find the approximate area of the region enclosed between the curve and the  $x$ -axis in the given domain.

$$\begin{aligned}
 A &= \frac{h}{2} [y_1 + y_n + 2(y_2 + y_3 + y_4 + \dots + y_{n-1})] \\
 &= \frac{\pi}{12} [0 + 0 + 2(0.5 + 0.866 + 1 + 0.866 + 0.5)] \\
 &= 1.95407
 \end{aligned}$$

- (c) Use integration to find the actual area of the region shown above.

$$\int_0^{\pi} \sin x dx = [-\cos x]_0^{\pi} = -[-1 - 1] = 2$$

- (d) Find the percentage error in your answer to (a) above.

$$\text{Percentage error} = \frac{2 - 1.95407}{2} \times 100 = 2.2965 = 2.3\%$$

Answer **all three** questions from this section.

**Question 7****(50 marks)**

A stadium can hold 25 000 people. People attending a regular event at the stadium must purchase a ticket in advance. When the ticket price is €20, the expected attendance at an event is 12 000 people. The results of a survey carried out by the owners suggest that for every €1 reduction, from €20, in the ticket price, the expected attendance would increase by 1000 people.

- (a) If the ticket price was €18, how many people would be expected to attend?

$$12000 + (20 - 18)1000 = 14000$$

- (b) Let  $x$  be the ticket price, where  $x \leq 20$ . Write down, in terms of  $x$ , the expected attendance at such an event.

$$12000 + (20 - x)1000 = 32000 - 1000x$$

- (c) Write down a function  $f$  that gives the expected income from the sale of tickets for such an event.

$$f(x) = (32000 - 1000x)x$$

- (d) Find the price at which tickets should be sold to give the maximum expected income.

$$f(x) = (32000 - 1000x)x$$

$$f'(x) = 32000 - 2000x = 0 \Rightarrow x = €16$$

- (e) Find this maximum expected income.

$$f(x) = (32000 - 1000x)x$$

$$f(16) = (32000 - 16000)16 = €256\,000$$



- (f) Suppose that tickets are instead priced at a value that is expected to give a full attendance at the stadium. Find the difference between the income from the sale of tickets at this price and the maximum income calculated at (e) above.

$$32000 - 1000x = 25000 \Rightarrow 1000x = 7000 \Rightarrow x = 7$$

$$f(x) = (32000 - 1000x)x \Rightarrow f(7) = (32000 - 7000)7 = 175\,000$$

$$\text{Difference: } \text{€}256\,000 - \text{€}175\,000 = \text{€}81\,000$$

- (g) The stadium was full for a recent special event. Two types of tickets were sold; a single ticket for €16 and a family ticket (2 adults and 2 children) for a certain amount. The income from this event was €365 000. If 1000 more family tickets had been sold the income from the event would have been reduced by €14 000. How many family tickets were sold?

Single ticket: €16; Family ticket € $y$

Number of single tickets:  $p$ ; Number of family tickets:  $\frac{25000-p}{4}$

$$16p + \frac{25000-p}{4}y = 365000$$

$$16(p - 4000) + \left(\frac{25000-p}{4} + 1000\right)y = 351000 \Rightarrow 16p + \frac{29000-p}{4}y = 415000$$

$$\frac{29000-p}{4}y - \frac{25000-p}{4}y = 50000 \Rightarrow 4000y = 200000 \Rightarrow y = 50$$

$$16p + \frac{25000-p}{4}50 = 365000 \Rightarrow 7p = 105000 \Rightarrow p = 15000$$

$$\text{Number of family tickets: } \frac{25000-p}{4} = \frac{25000-15000}{4} = 2500$$

**OR**

$x$  = number of single tickets

$f$  = number of family tickets

$y$  = cost of family ticket

$$x + 4f = 25000$$

$$16x + fy = 365000$$

$$16(x - 4000) + (f + 1000)y = 351000$$

$$16x - 64000 + fy + 1000y = 351000$$

$$16x \quad + \quad fy \quad = 365000$$

$$\hline 1000y = 50000$$

$$y = 50$$

$$x + 4f = 25000$$

$$16x + 50f = 365000$$

$$16x + 64f = 400000$$

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$$14f = 35000$$

$$f = 2500$$

**Question 8****(50 marks)**

The speed at which a raindrop falls increases until a maximum speed, called its *terminal velocity*, is reached. The raindrop then continues to fall at this terminal velocity.

The distance,  $s$  metres, it falls is given by

$$s(t) = \begin{cases} 6t + 0.3t^2 - 0.01t^3, & 0 \leq t \leq 10 \\ k(t-10), & t > 10 \end{cases}$$

where  $t$  is the time in seconds from the instant the raindrop begins to fall and  $k$  is a constant.

- (a) How far has this raindrop fallen after 10 seconds?

$$s(t) = 6t + 0.3t^2 - 0.01t^3$$

$$s(10) = 6(10) + 0.3(10)^2 - 0.01(10)^3 = 80 \text{ m}$$

- (b) After how many seconds is the raindrop falling at a speed of 8.25 metres per second?

$$s(t) = 6t + 0.3t^2 - 0.01t^3$$

$$\frac{ds}{dt} = 6 + 0.6t - 0.03t^2 = 8.25$$

$$\Rightarrow 600 + 60t - 3t^2 = 825$$

$$\Rightarrow t^2 - 20t + 75 = 0$$

$$\Rightarrow (t-5)(t-15) = 0$$

$$\Rightarrow t = 5 \text{ or } t = 15$$

$$0 \leq t \leq 10 \Rightarrow t = 5$$

- (c) The acceleration of the raindrop is decreasing for the first 10 seconds of its fall. Find the value of  $t$  for which the acceleration is  $0.006 \text{ m s}^{-2}$ .

$$\frac{d^2s}{dt^2} = 0.6 - 0.06t = 0.006 \Rightarrow t = 9.9 \text{ s}$$

- (d) The raindrop falls vertically from a height of 620 metres. How long will it take the raindrop to fall to ground level?

$$t = 10 \Rightarrow s = 80 \text{ m}$$
$$t > 10 \Rightarrow s = 620 - 80 = 540 \text{ m}$$

$$\frac{ds}{dt} = 6 + 0.6t - 0.03t^2$$
$$= 6 + 0.6(10) - 0.03(10)^2$$
$$= 9 \text{ m s}^{-1}$$

$$\frac{540}{9} = 60 \text{ s}$$

$$\text{Total: } 10 + 60 = 70 \text{ s}$$

- (e) A raindrop increases in size as it falls. The volume of a spherical raindrop increases at a rate of 6 cubic millimetres per second. Find the rate at which the radius of the raindrop is increasing when the radius is 1.5 mm.

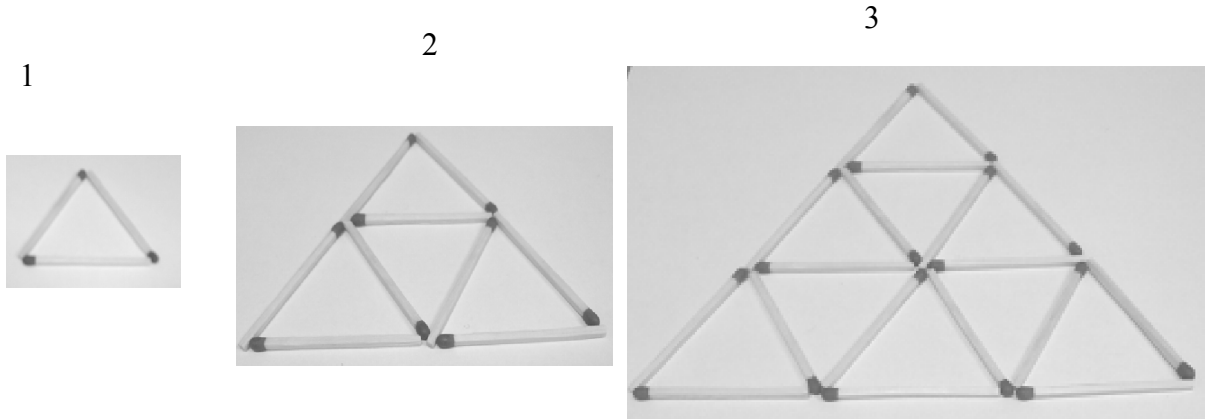
$$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2$$

$$\frac{dV}{dt} = \frac{dV}{dr} \frac{dr}{dt} \Rightarrow 6 = 4\pi r^2 \frac{dr}{dt} = 4\pi(2.25) \frac{dr}{dt}$$
$$\Rightarrow \frac{dr}{dt} = \frac{6}{9\pi} = 0.2122 \text{ mm s}^{-1}$$

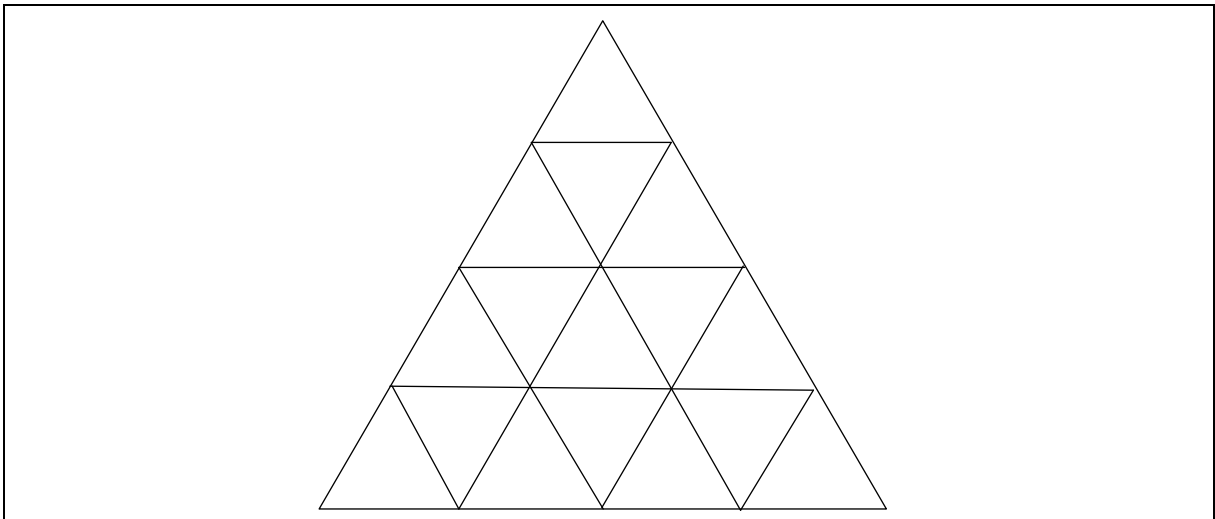
**Question 9**

**(50 marks)**

Shapes in the form of small equilateral triangles can be made using matchsticks of equal length. These shapes can be put together into patterns. The beginning of a sequence of these patterns is shown below.



(a) (i) Draw the 4<sup>th</sup> pattern in the sequence.



(ii) The table below shows the number of small triangles in each pattern and the number of matchsticks needed to create each pattern. Complete the table.

Pattern	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Number of small triangles	1	4	9	16
Number of matchsticks	3	9	18	30

- (b) Write an expression in  $n$  for the number of triangles in the  $n^{\text{th}}$  pattern in the sequence.

$n^2$

**OR**

1                      4                      9                      16                      25

1<sup>st</sup> Diff:    3                      5                      7                      9

2<sup>nd</sup> Diff:                      2                      2                      2    ← Constant ⇒ quadratic

$$T_n = an^2 + bn + c$$

$$2a = 2 \Rightarrow a = 1$$
  

$$T_n = n^2 + bn + c$$

$$T_1 = 1 + b + c = 1$$

$$T_2 = 4 + 2b + c = 4$$

$$\Rightarrow b = 0$$

$$c = 0$$
  

$$\Rightarrow T_n = n^2$$

- (c) Find an expression, in  $n$ , for the number of matchsticks needed to turn the  $(n-1)^{\text{th}}$  pattern into the  $n^{\text{th}}$  pattern.

$3n$

**OR**

3                      9                      18                      30

1<sup>st</sup> Diff:    6                      9                      12

2<sup>nd</sup> Diff:                      3                      3

Second difference constant ⇒ quadratic pattern

$$T_n = an^2 + bn + c$$

$$2a = 3 \Rightarrow a = \frac{3}{2}$$

$$T_n = \frac{3}{2}n^2 + bn + c$$

$$T_1 = \frac{3}{2}(1)^2 + b(1) + c = 3 \Rightarrow b + c = \frac{3}{2}$$

$$T_2 = \frac{3}{2}(2)^2 + b(2) + c = 9 \Rightarrow 2b + c = 3$$

$$b + c = \frac{3}{2}$$

$$\underline{2b + c = 3}$$

$$b = \frac{3}{2}$$

$$c = 0$$

$$T_n = \frac{3}{2}n^2 + \frac{3}{2}n$$

$$T_{n-1} = \frac{3}{2}(n-1)^2 + \frac{3}{2}(n-1)$$

$$T_n - T_{n-1} = -\frac{3}{2}(n^2 - 2n + 1) - \frac{3}{2}(n-1) + \frac{3}{2}n^2 + \frac{3}{2}n$$

$$= -\frac{3}{2}n^2 + 3n - \frac{3}{2} - \frac{3}{2}n + \frac{3}{2} + \frac{3}{2}n^2 + \frac{3}{2}n$$

$$= 3n$$

- (d) The number of matchsticks in the  $n^{\text{th}}$  pattern in the sequence can be represented by the function  $u_n = an^2 + bn$  where  $a, b \in \mathbb{Q}$  and  $n \in \mathbb{N}$ . Find the value of  $a$  and the value of  $b$ .

$$u_n = an^2 + bn$$

$$u_1 = a(1)^2 + b(1) = 3$$

$$u_2 = a(2)^2 + b(2) = 9$$

$$\begin{array}{rcl} a + b = 3 & \Rightarrow & 2a + 2b = 6 \\ 4a + 2b = 9 & \Rightarrow & \underline{4a + 2b = 9} \\ & & 2a = 3 \Rightarrow a = \frac{3}{2} \end{array}$$

$$a + b = 3 \Rightarrow \frac{3}{2} + b = 3 \Rightarrow b = \frac{3}{2}$$

**OR**

3, 9, 18, 30,.....

= 3, 3+6, 3+6+9, 3+6+9+12,.....

=  $S_1, S_2, S_3, S_4, \dots$  of 3+6+9+12+.....

$$S_n = \frac{n}{2}[2a + (n-1)d], \text{ where } a = d = 3$$

$$\Rightarrow S_n = \frac{n}{2}[6 + (n-1)3]$$

$$\Rightarrow S_n = \frac{3n}{2}(n+1)$$

Equating  $an^2 + bn$  and  $\frac{3n}{2}(n+1)$ ,

$$\Rightarrow a = b = \frac{3}{2}$$



- (e) One of the patterns in the sequence has 4134 matchsticks. How many small triangles are in that pattern?

$$u_n = \frac{3}{2}n^2 + \frac{3}{2}n = 4234$$

$$\Rightarrow n^2 + n - 2756$$

$$\Rightarrow (n+53)(n-52) = 0$$

$$\Rightarrow n = -53 \text{ or } n = 52.$$

$$n^2 = 52^2 = 2704 \text{ triangles}$$

**OR**

From (d):  $u_n = \frac{3}{2}n^2 + \frac{3}{2}n = 4134$

$$3n^2 + 3n = 8268$$

$$n^2 + n - 2756 = 0$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1 - 4(1)(-2756)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{11025}}{2}$$

$$= \frac{-1 \pm 105}{2}$$

$$= -53 \text{ or } 52$$

$$\Rightarrow n = 52$$

$$n^2 = 52^2 = 2704 \text{ triangles}$$

## Marking Scheme – Paper 1, Section A and Section B

### Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	B	C	D
No of categories	3	4	5
5 mark scales	0, 2, 5	0, 2, 4, 5	0, 2, 3, 4, 5
10 mark scales	0, 5, 10	0, 3, 7, 10	0, 3, 5, 8, 10
15 mark scales	0, 7, 15	0, 5, 10, 15	0, 4, 7, 11, 15

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

### Marking scales – level descriptors

#### B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

#### C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

#### D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, *scale 10C\** indicates that 9 marks may be awarded.

## Summary of mark allocations and scales to be applied

### *Section A*

Question 1	
(a)	10C
(b)	10C
(c)	5C
Question 2	
(a)	10C
(b)	15D
Question 3	
(a)	15B
(b)	10C*
Question 4	
(a)(i)	5C*
(a)(ii)	10D*
(b)	10C*
Question 5	
(a)	15D
(b)	10B
Question 6	
(a)	5C*
(b)	10C
(c)	5C
(d)	5B

### *Section B*

Question 7	
(a)	10B
(b)	5B
(c)	5B
(d)	10B
(e)	5B
(f)	10C
(g)	5D
Question 8	
(a)	10C*
(b)	10D*
(c)	10C*
(d)	10D*
(e)	10D*
Question 9	
(a)(i)	5B
(a)(ii)	5C
(b)	10B
(c)	10B
(d)	10C
(e)	10C

## Detailed marking notes

### Section A

#### Question 1

- (a) Scale 10C (0, 3, 7, 10)  
*Low Partial Credit:*
- Does not multiply by conjugate
  - Drops  $i$ , or  $i^2 \neq -1$
  - Incomplete cross-multiplication

*High Partial Credit:*

- Work not simplified

- (b) Scale 10C (0, 3, 7, 10).

*Low Partial Credit:*

- Work with  $\alpha$
- Work with  $\theta$
- Work with modulus
- Plotting  $z$

*High Partial Credit:*

- Correct  $z$  but incorrect or no plotting

**Note:** Accept  $r$ ,  $\theta$  and plot for full marks.

- (c) Scale 5C (0, 2, 4, 5).

*Low Partial Credit:*

- Some work with De Moivre
- De Moivre not used correctly

*High Partial Credit:*

- Answer not simplified
- $n$  included in answer

**Note:** Allow for full marks candidates incorrect angle from (b), with correct conclusion.

**0:** no use of De Moivre.

## Question 2

(a) Scale 10C (0, 3, 7, 10)

*Low Partial Credit:*

- Factorises
- Gets roots
- Some use of quadratic root formula

*High Partial Credit:*

- Wrong shape of graph, but otherwise correct
- Incorrect deduction for correct values of  $x$
- Correct shading on  $x$ -axis
- Using  $x >$  only

(b) Scale 15D (0, 4, 7, 11, 15)

*Low Partial Credit:*

- Any relevant step to solution

*Mid Partial Credit:*

- Reduces to two unknowns correctly in one equation

*High Partial Credit:*

- Evaluates one unknown

## Question 3

(a) Scale 15B (0, 7, 15)

*Partial Credit:*

- Value correctly substituted into  $e$

(b) Scale 10C\* (0, 3, 7, 10)

*Low Partial Credit:*

- Correct statement
- Uses logs (correctly or incorrectly)
- Makes an effort to isolate  $t$

*High Partial Credit:*

- Gets correct linear equation for  $t$

**Note:** Incomplete rounding or incorrect rounding or no rounding gets 9 marks.

#### Question 4

(a)(i) Scale 5C\* (0, 2, 4, 5)

*Low Partial Credit:*

- Any relevant first step
- Correct statement with no work

*High Partial Credit:*

- $r$  not as a %

**Note:** Incomplete rounding or incorrect rounding or no rounding gets 4 marks.

(a)(ii) Scale 10D\* (0, 3, 5, 8, 10)

*Low Partial Credit:*

- Any relevant step
- Reference to 1.00327

*Mid Partial Credit:*

- Recognises G.P.

*High Partial Credit:*

- Expression for sum of G.P.

**Note:** Incomplete rounding or incorrect rounding or no rounding gets 9 marks.

**OR**

(a)(ii) Scale 10D\* (0, 3, 5, 8, 10)

*Low Partial Credit:*

- No present value or incorrect present value

*Mid Partial Credit:*

- Present value correct

*High Partial Credit:*

- Correct substitution of all values in formula

**Note:** Incomplete rounding or incorrect rounding or no rounding gets 9 marks.

(b) Scale 10C\* (0, 3, 7, 10)

*Low Partial Credit:*

- Any relevant step

*High Partial Credit:*

- Correct substitution of all values in formula
- Expression for sum of G.P. in solution

**Note:** Incomplete rounding or incorrect rounding or no rounding gets 9 marks.

### Question 5

(a) Scale 15D (0, 4, 7, 11, 15)

*Low Partial Credit:*

- One or two correct entries
- Some correct work with candidates own choice of functions

*Mid Partial Credit:*

- Three to five correct entries
- Six to ten correct entries in candidate's own choice of particular quadratic, cubic, trigonometric and exponential functions

*High Partial Credit:*

- Six to ten correct entries
- Candidate chooses particular quadratic, cubic, trigonometric and exponential functions, with correct first and second derivatives

(b) Scale 10B (0, 5, 10).

*Partial Credit:*

- Some correct statement regarding the first or second derivative of a function
- Correct explanation of either first or second derivative

## Question 6

(a) Scale 5C\* (0, 2, 4, 5)

*Low Partial Credit:*

- One or two correct entries

*High Partial Credit:*

- Three to six correct entries

(b) Scale 10C (0, 3, 7, 10).

*Low Partial Credit:*

- Correct formula.
- Substitutes in the formula, values from the domain of the function

*High Partial Credit:*

- Correct substitution in the formula and stops
- Incorrect  $\frac{h}{2}$  and finishes correctly

(c) Scale 5C (0, 2, 4, 5).

*Low Partial Credit:*

- Correct limits
- 'c' appears and no other correct work

*High Partial Credit:*

- Correct integration

(d) Scale 5B (0, 2, 5)

*Partial Credit:*

- Any correct relevant step



## Section B

### Question 7

(a) Scale 10B (0, 5, 10)

*Partial Credit:*

- $(20-18)1000$  or equivalent

(b) Scale 5B (0, 2, 5)

*Partial Credit:*

- Expression  $(20 - x)$

**Note:** Accept for 5 marks  $12000 + (20 - x)1000$  or equivalent.

(c) Scale 5B (0, 2, 5)

*Partial Credit:*

- Correct number of people and/or correct rate in terms of  $x$

(d) Scale 10B (0, 5, 10)

*Partial Credit:*

- Some correct differentiation of a quadratic function
- $(32000 - 2000x) = 0$  or equivalent
- Correct testing with incorrect deduction or no deduction
- Possible to get full marks without use of calculus
- Correct answer and no work

(e) Scale 5B (0, 2, 5)

*Partial Credit:*

- Some effort at substitution of 16 or equivalent

(f) Scale 10C (0, 3, 7, 10)

*Low Partial Credit:*

- Use of expression
- Use of 25000
- Some use of tables
- Equation solved
- Price of ticket found

*High Partial Credit:*

- Total income from sales

(g) Scale 5D (0, 2, 3, 4, 5)

*Low Partial Credit:*

- Correct number of family tickets
- One equation only
- Income from single tickets
- Income from family tickets

*Mid Partial Credit:*

- Two correct linear equations for income in two unknowns
- $y = 50$  without work, or  $p = 15000$  without work

*High Partial Credit:*

- Correct value for  $p$  (single ticket)
- Correct value for  $y$  (family ticket)

**OR**

(g) Scale 5D (0, 2, 3, 4, 5)

*Low Partial Credit:*

- One correct equation

*Mid Partial Credit:*

- Three correct equations

*High Partial Credit:*

- One unknown calculated

## Question 8

(a) Scale 10C\* (0, 3, 7, 10)

*Low Partial Credit:*

- Some correct substitution into  $s(t)$

*High Partial Credit:*

- Correct substitution.
- $10s = 80 \Rightarrow s = 8$

*Full Marks:* Correct answer without work

(b) Scale 10D\* (0, 3, 5, 8, 10)

*Low Partial Credit:*

- Any correct differentiation

*Mid Partial Credit:*

- $\frac{ds}{dt}$  found correctly

*High Partial Credit:*

- Fails to select correct value for  $t$

(c) Scale 10C\* (0, 3, 7, 10)

*Low Partial Credit:*

- Some correct differentiation of the first derivative

*High Partial Credit:*

- Sets up correct equation.

(d) Scale 10D\* (0, 3, 5, 8, 10)

*Low Partial Credit:*

- Some correct substitution
- Finds distance travelled after initial 10 seconds i.e. 540m
- Correct differentiation (relevant for terminal velocity)

*Mid Partial Credit:*

- Finds  $k$ .

*High Partial Credit:*

- Finds time travelling at terminal velocity and stops

(e) Scale 10D\* (0, 3, 5, 8, 10)

*Low Partial Credit:*

- $V = \frac{4}{3}\pi r^3$
- Any relevant step

*Mid Partial Credit:*

- Any two of  $\frac{dv}{dt} = 6$ ,  $\frac{dv}{dr} = 4\pi r^2$ ,  $\frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}$

*High Partial Credit:*

- Correct substitution into chain rule
- Fails to substitute for  $r$

### Question 9

(a)(i) Scale 5B (0, 2, 5)

*Partial Credit:*

- Incomplete 4<sup>th</sup> line

(a)(ii) Scale 5C (0, 2, 4, 5)

*Low Partial Credit:*

- One or two correct entries in empty boxes

*High Partial Credit:*

- Three correct entries in empty boxes

(b) Scale 10B (0, 5, 10)

*Partial Credit:*

- $1^2, 2^2, 3^2$  etc – recognising the natural numbers squared

**OR**

(b) Scale 10B (0, 5, 10)

*Partial Credit:*

- Second differences calculated

(c) Scale 10B (0, 5, 10)

*Partial Credit:*

- Recognition of series 6, 9, 12,.... or similar

**OR**

(c) Scale 10B (0, 5, 10)

*Partial Credit:*

- Second differences calculated

- (d) Scale 10C (0, 3, 7, 10)  
*Low Partial Credit:*
- One linear equation in  $a$  and  $b$ , e.g.  $u_1: a + b = 3$
- High Partial Credit:*
- Two correct linear equations

**OR**

- (d) Scale 10C (0, 3, 7, 10)  
*Low Partial Credit:*
- Recognition of A.P.
- High Partial Credit:*
- $a = d = 3$  and some use of  $S_n$  formula
- (e) Scale 10C (0, 3, 7, 10)  
*Low Partial Credit:*
- Expression of  $u_n$  in one variable only
  - Quadratic equation
- High Partial Credit:*
- Values of  $n$



Coimisiún na Scrúduithe Stáit  
State Examinations Commission

Leaving Certificate Examination 2013

Mathematics  
(Project Maths – Phase 3)

Paper 2

Higher Level

Monday 10 June      Morning 9:30 – 12:00

300 marks

## Model Solutions – Paper 2

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.

## Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions, as follows:

In Section A, answer:

Questions 1 to 5 and

**either** Question 6A **or** Question 6B.

In Section B, answer Questions 7 to 9.

Write your answers in the spaces provided in this booklet. You will lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:



Answer all six questions from this section.

**Question 1**

**(25 marks)**

(a) Explain each of the following terms:

(i) Sample space

The set of all possible outcomes of an experiment.

(ii) Mutually exclusive events

Events E and F are mutually exclusive if they have no outcomes in common.  
i.e  $P(E \cup F) = P(E) + P(F)$

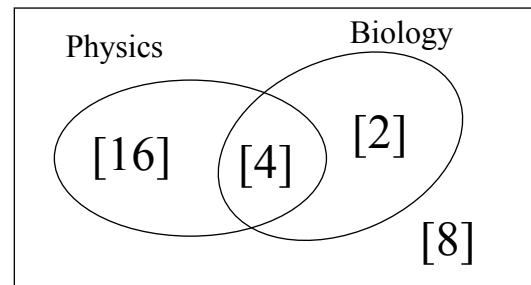
(iii) Independent events

Two events are independent if the outcome of one does not depend on the outcome of the other.

i.e  $P(E \cap F) = P(E) \cdot P(F)$  or  $P(E|F) = P(E)$  or  $P(F|E) = P(F)$

(b) In a class of 30 students, 20 study Physics, 6 study Biology and 4 study both Physics and Biology.

(i) Represent the information on the Venn Diagram.



A student is selected at random from this class.

The events E and F are:

E: The student studies Physics

F: The student studies Biology.

(ii) By calculating probabilities, investigate if the events E and F are independent.

$$P(E \cap F) = \frac{4}{30}$$

$$P(E) \times P(F) = \frac{20}{30} \times \frac{6}{30} = \frac{4}{30}$$

$$P(E \cap F) = P(E) \times P(F) \Rightarrow \text{E and F are independent events}$$

**Question 2****(25 marks)****(a)** A random variable  $X$  follows a normal distribution with mean 60 and standard deviation 5.**(i)** Find  $P(X \leq 68)$ .

$$P(X \leq 68) = P\left(Z \leq \frac{68 - 60}{5}\right) = P(Z \leq 1.6) = 0.9452$$

**(ii)** Find  $P(52 \leq X \leq 68)$ .

$$P(52 \leq X \leq 68) = P\left(\frac{52 - 60}{5} \leq Z \leq \frac{68 - 60}{5}\right)$$

$$= P(-1.6 \leq Z \leq 1.6)$$

$$P(Z \leq -1.6) = P(Z \geq 1.6)$$

$$= 1 - P(Z \leq 1.6)$$

$$= 1 - 0.9452 = 0.0548$$

$$P(-1.6 \leq Z \leq 1.6) = P(Z \leq 1.6) - P(Z \leq -1.6)$$

$$= 0.9452 - 0.0548 = 0.8904$$

**OR**

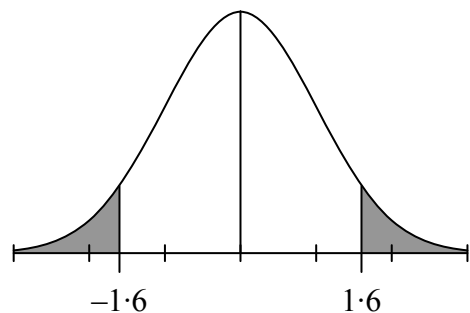
$$P(52 \leq X \leq 68) = P\left(\frac{52 - 60}{5} \leq Z \leq \frac{68 - 60}{5}\right)$$

$$= P(-1.6 \leq Z \leq 1.6)$$

$$= 1 - 2P(Z \geq 1.6)$$

$$= 1 - 2(1 - P(Z \leq 1.6))$$

$$= 1 - 2(1 - 0.9452) = 1 - 2(0.0548) = 1 - 0.1096 = 0.8904$$

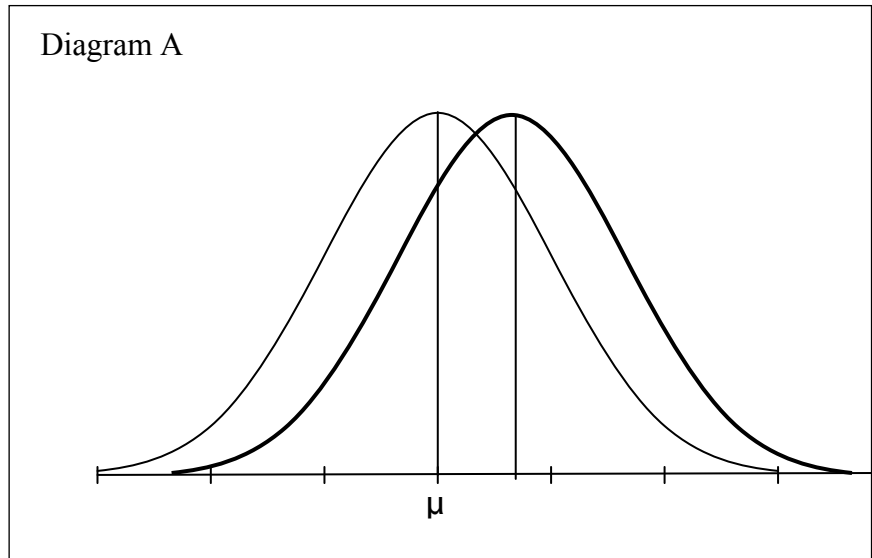
**(b)** The heights of a certain type of plant, when ready to harvest, are known to be normally distributed, with a mean of  $\mu$ . A company tests the effects of three different types of growth hormone on this type of plant. The three hormones were used on different large samples of the crop. After applying each hormone, it was found that the heights of the plants in the samples were still normally distributed at harvest time.

The diagrams A, B and C show the expected distribution of the heights of the plants, at harvest time, without the use of the hormones.

The effect, on plant growth, of each of the hormones is described. Sketch, on each diagram, a new distribution to show the effect of the hormone.

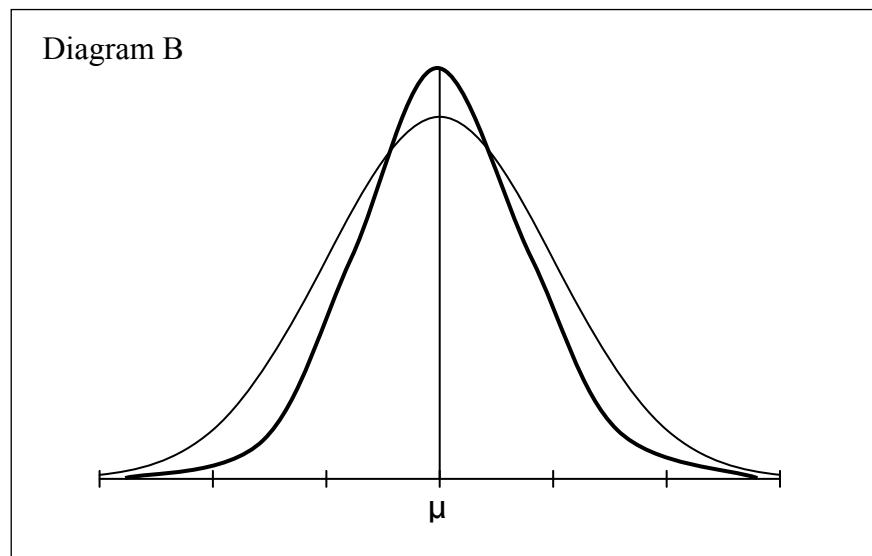
Hormone A

The effect of hormone A was to increase the height of all of the plants.



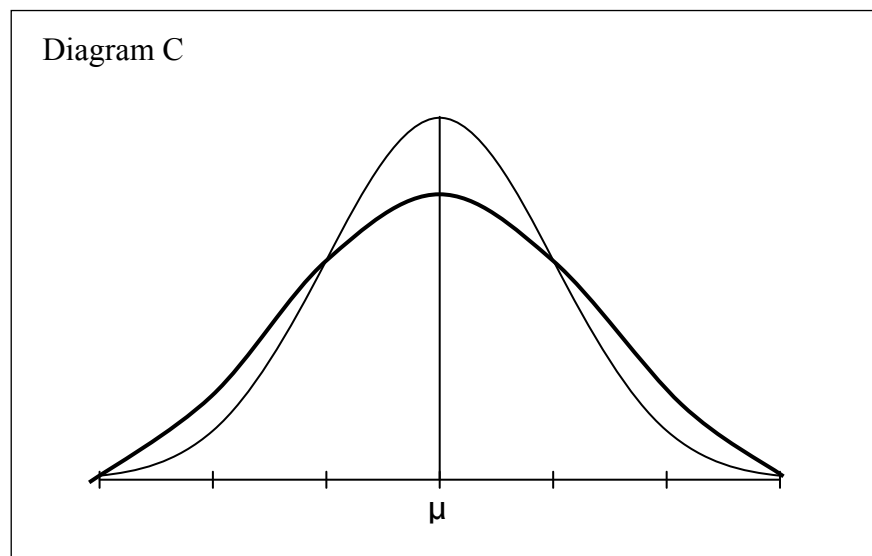
Hormone B

The effect of hormone B was to reduce the number of really small plants and the number of really tall plants. The mean was unchanged.



Hormone C

The effect of hormone C was to increase the number of small plants and the number of tall plants. The mean was unchanged.



**Question 3**

**(25 marks)**

The equations of six lines are given:

Line	Equation
<i>h</i>	$x = 3 - y$
<i>i</i>	$2x - 4y = 3$
<i>k</i>	$y = -\frac{1}{4}(2x - 7)$
<i>l</i>	$4x - 2y - 5 = 0$
<i>m</i>	$x + \sqrt{3}y - 10 = 0$
<i>n</i>	$\sqrt{3}x + y - 10 = 0$

(a) Complete the table below by matching each description given to one or more of the lines.

Description	Line(s)
A line with a slope of 2.	<i>l</i>
A line which intersects the <i>y</i> -axis at $(0, -2\frac{1}{2})$ .	<i>l</i>
A line which makes equal intercepts on the axes.	<i>h</i>
A line which makes an angle of $150^\circ$ with the positive sense of the <i>x</i> -axis.	<i>m</i>
Two lines which are perpendicular to each other.	<i>l</i> and <i>k</i>

(b) Find the acute angle between the lines *m* and *n*.

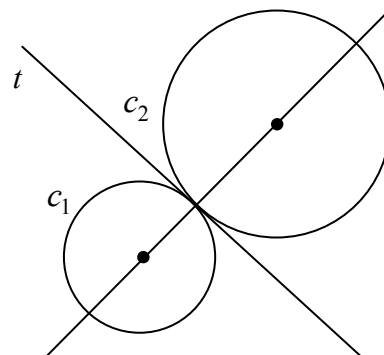
<p>Slope of <i>m</i>: <math>m_1 = -\frac{1}{\sqrt{3}}</math></p> <p>Slope of <i>n</i>: <math>m_2 = -\sqrt{3}</math></p> $\tan \theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2} = \pm \frac{-\frac{1}{\sqrt{3}} + \sqrt{3}}{1 - \frac{1}{\sqrt{3}}(-\sqrt{3})} = \pm \frac{-1 + 3}{\sqrt{3} + 1} = \pm \frac{1}{\sqrt{3}}$ <p><math>\tan \theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ</math></p>
--

**Question 4**

**(25 marks)**

The circles  $c_1$  and  $c_2$  touch externally as shown.

$$\sqrt{g^2 + f^2 - c} = \sqrt{1+1+7} = 3$$



(a) Complete the following table:

Circle	Centre	Radius	Equation
$c_1$	$(-3, -2)$	2	$(x+3)^2 + (y+2)^2 = 4$ <b>OR</b> $x^2 + y^2 + 6x + 4y + 9 = 0$
$c_2$	$(1, 1)$	3	$x^2 + y^2 - 2x - 2y - 7 = 0$

(b) (i) Find the co-ordinates of the point of contact of  $c_1$  and  $c_2$ .

Divide line segment joining  $(-3, -2)$  and  $(1, 1)$  in ratio 2 : 3

$$\left( \frac{2(1) + 3(-3)}{2+3}, \frac{2(1) + 3(-2)}{2+3} \right) = \left( -\frac{7}{5}, -\frac{4}{5} \right)$$

**OR**

Slope line of centres =  $\frac{3}{4}$ .

Equation line of centres:  $y - 1 = \frac{3}{4}(x - 1) \Rightarrow 3x - 4y + 1 = 0$

$c_1 - c_2 = 4x + 3y + 8 = 0$

$4x + 3y + 8 = 0 \cap 3x - 4y + 1 = 0 \Rightarrow x = -\frac{7}{5}, y = -\frac{4}{5}$

(ii) Hence, or otherwise, find the equation of the tangent,  $t$ , common to  $c_1$  and  $c_2$ .

$$\text{Slope of line of centres: } \frac{1+2}{1+3} = \frac{3}{4}$$

$$\text{Slope of tangent: } m = -\frac{4}{3}$$

$$\begin{aligned}\text{Equation of tangent: } y + \frac{4}{5} &= -\frac{4}{3}\left(x + \frac{7}{5}\right) \\ \Rightarrow 3y + \frac{12}{5} &= -4x - \frac{28}{5} \\ \Rightarrow 4x + 3y + 8 &= 0\end{aligned}$$

**OR**

$$\begin{aligned}c_1 - c_2 &= x^2 + y^2 + 6x + 4y + 9 - (x^2 + y^2 - 2x - 2y - 7) = 0 \\ \Rightarrow 6x + 4y + 9 - (-2x - 2y - 7) &= 0 \\ \Rightarrow 8x + 6y + 16 = 0 &\Rightarrow 4x + 3y + 8 = 0\end{aligned}$$

**OR**

$$\begin{aligned}xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c &= 0 \\ x\left(-\frac{7}{5}\right) + y\left(-\frac{4}{5}\right) + 3\left(x + \left(-\frac{7}{5}\right)\right) + 2\left(y + \left(-\frac{4}{5}\right)\right) + 9 &= 0 \\ \Rightarrow 4x + 3y + 8 &= 0\end{aligned}$$

**Question 5**

**(25 marks)**

- (a) In a triangle  $ABC$ , the lengths of the sides are  $a$ ,  $b$  and  $c$ . Using a formula for the area of a triangle, or otherwise, prove that

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}.$$

$$\frac{1}{2}ac \sin \angle B = \frac{1}{2}ab \sin \angle C$$

Divide by  $\frac{1}{2}abc$

$$\frac{\sin \angle B}{b} = \frac{\sin \angle C}{c} \Rightarrow \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$$

**OR**

**Case 1**

$$\sin \angle B = \frac{x}{c}$$

$$x = c \sin \angle B$$

$$b \sin \angle C = c \sin \angle B$$

$$\frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$$

$$\sin \angle C = \frac{x}{b}$$

$$x = b \sin \angle C$$

**Case 2**

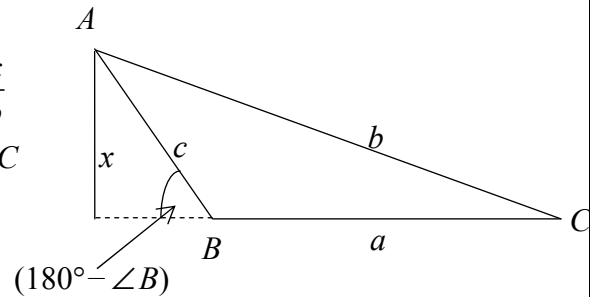
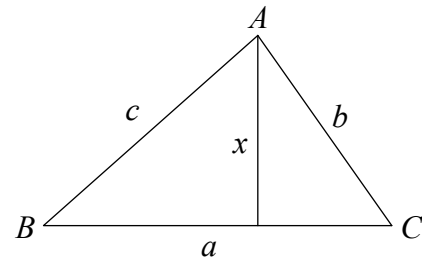
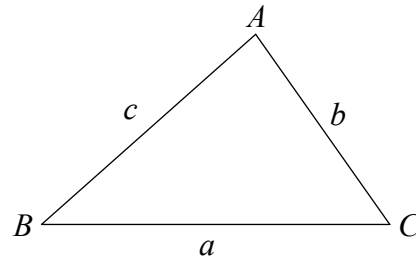
$$\sin(180^\circ - \angle B) = \frac{x}{c}$$

$$x = c \sin(180^\circ - \angle B)$$

$$x = c \sin \angle B$$

$$b \sin \angle C = c \sin \angle B$$

$$\frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$$



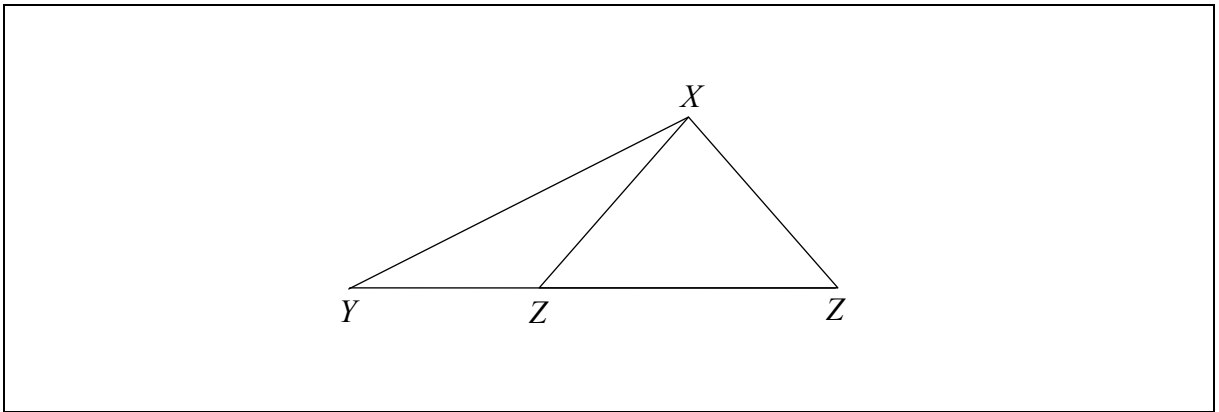
- (b) In a triangle  $XYZ$ ,  $|XY| = 5$  cm,  $|XZ| = 3$  cm and  $|\angle XYZ| = 27^\circ$ .

- (i) Find the two possible values of  $|\angle XZY|$ . Give your answers correct to the nearest degree.

$$\frac{3}{\sin 27^\circ} = \frac{5}{\sin \angle Z} \Rightarrow \sin \angle Z = \frac{5 \sin 27^\circ}{3} = 0.756$$

$$\Rightarrow |\angle Z| = 49^\circ \quad \text{or} \quad |\angle Z| = 131^\circ$$

- (ii) Draw a sketch of the triangle  $XYZ$ , showing the two possible positions of the point  $Z$ .



- (c) In the case that  $|\angle XZY| < 90^\circ$ , write down  $|\angle ZXY|$ , and hence find the area of the triangle  $XYZ$ , correct to the nearest integer.

$$|\angle ZXY| = 180^\circ - (27^\circ + 49^\circ) = 104^\circ$$

$$\Delta = \frac{1}{2}ab \sin C = \frac{1}{2}(5)(3) \sin 104^\circ = 7.27 = 7 \text{ cm}^2$$



**Question 6**

**(25 marks)**

Answer **either** 6A **or** 6B.

**Question 6A**

(a) Complete each of the following statements.

(i) The circumcentre of a triangle is the point of intersection of

the perpendicular bisectors of the sides of the triangle

(ii) The incentre of a triangle is the point of intersection of

the bisectors of the angles of the triangle

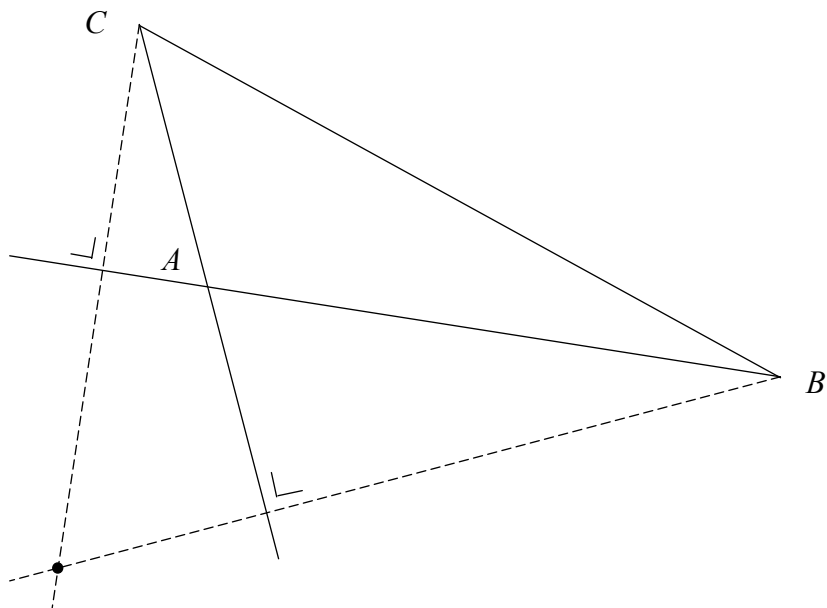
(iii) The centroid is the point of intersection of

the medians of the triangle

(b) In an equilateral triangle, the circumcentre, the incentre and the centroid are all in the same place. Explain why this is the case.

In an equilateral triangle the medians are perpendicular to the opposite sides and bisect the angles. Therefore, the perpendicular bisectors of the sides, the bisectors of the angles and the median are the same line and intersect at one point.

(c) Construct the orthocentre of the triangle  $ABC$  below. Show all construction lines clearly.



OR

Question 6B

- (a) A quadrilateral (four sided figure) has two sides which are parallel and equal in length.  
Prove that the quadrilateral is a parallelogram.

In the quadrilateral  $WXYZ$ ,  $WX \parallel ZY$  and  $|WX| = |ZY|$

To Prove:  $WXYZ$  is a parallelogram.

Join  $Z$  to  $X$  and  $Y$  to  $W$

Proof:

In  $\triangle ZOY$  and  $\triangle OWX$ ,

$$|ZY| = |WX|$$

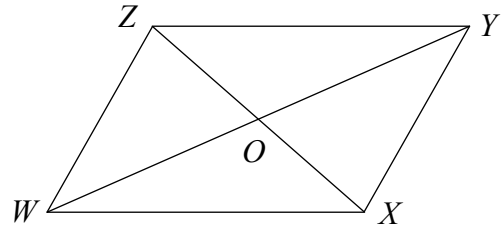
$$|\angle ZYO| = |\angle OWX| \dots ZY \parallel WX$$

$$|\angle YZO| = |\angle OXW| \dots ZY \parallel WX$$

Hence,  $\triangle ZOY$  and  $\triangle OWX$  are congruent since AAS

$$\text{Hence, } |ZO| = |OX| \text{ and } |YO| = |OW|$$

Hence, the diagonals of  $WXYZ$  bisect each other  $\Rightarrow WXYZ$  is a parallelogram.



OR

In the quadrilateral  $WXYZ$ ,  $WX \parallel ZY$  and  $|WX| = |ZY|$

To Prove:  $WXYZ$  is a parallelogram.

Join  $Z$  to  $X$

Proof:

In  $\triangle WXZ$  and  $\triangle YZX$ ,

$$|WX| = |ZY|$$

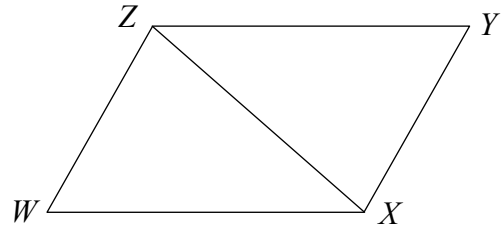
$$|\angle YZX| = |\angle WXZ| \dots ZY \parallel WX$$

$$|ZX| = |ZX| \dots \text{common to both}$$

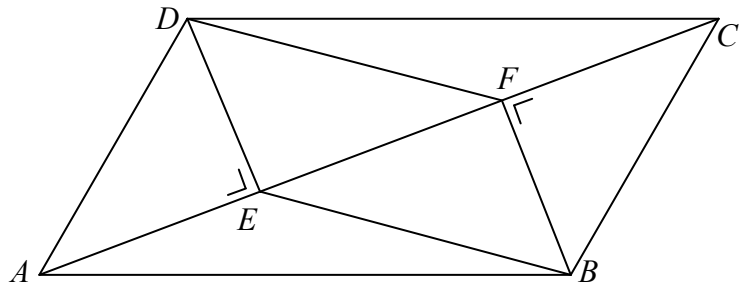
Hence,  $\triangle WXZ$  and  $\triangle YZX$  are congruent since SAS

$$\Rightarrow WZ \text{ and } XY \text{ parallel}$$

$\Rightarrow WXYZ$  is a parallelogram.



- (b) In the parallelogram  $ABCD$ ,  
 $DE$  is perpendicular to  $AC$ .  
 $BF$  is perpendicular to  $AC$ .  
 Prove that  $EBFD$  is a parallelogram.



In the parallelogram  $ABCD$ ,  
 $DE \perp AC$  and  $AC \perp BF \Rightarrow DE \parallel BF$ .

In the parallelogram  $ABCD$ ,  
 area of  $\triangle DAC = \text{area of } \triangle ABC \Rightarrow |DE| = |BF|$ .

$DE \parallel BF$  and  $|DE| = |BF| \Rightarrow EBFD$  is a parallelogram.

**Section B****Contexts and Applications****150 marks**

Answer **all three** questions from this section.

**Question 7****(75 marks)**

*Go Fast Airlines* provides internal flights in Ireland, short haul flights to Europe and long haul flights to America and Asia. On long haul flights the company sells economy class, business class and executive class tickets. All passengers have a baggage allowance of 20 kg and must pay a cost per kg for any weight over the 20 kg allowance.

Each month the company carries out a survey among 1000 passengers. Some of the results of the survey for May are shown below.

Gender	Male: 479	Female: 521
--------	-----------	-------------

Previously flown with <i>Go Fast Airlines</i>	Yes: 682	No: 318
Would fly again with <i>Go Fast Airlines</i>	Yes: 913	No: 87

Passenger Age	Mean age: 42
	Median age: 31

Spend on in-flight facilities	Mean spend: €18.65
	Median spend: €32.18

Was flight delayed	Yes	No	Don't Know
	231	748	21

Passenger satisfaction with overall service	Satisfied	Not satisfied	Don't Know
	664	238	98

(a) *Go Fast Airlines* used a **stratified random sample** to conduct the survey.

(i) Explain what is meant by a **stratified random sample**.

The population is divided into different subgroups which have common characteristics. Random samples are drawn from each subgroup according to their proportion of the population.

- (ii) Write down 4 different passenger groups that the company might have included in their sample.

One solution:

Long haul economy class passengers.  
 Long haul business class passengers.  
 Long haul executive class passengers.  
 Short haul passengers

- (b) (i) What is the probability that a passenger selected at random from this sample

- had their flight delayed

$$\frac{231}{1000} = 0.231 \text{ or } \frac{231}{979} = 0.236$$

- Was not satisfied with the overall service

$$\frac{238}{1000} = 0.238 \text{ or } \frac{238}{902} = 0.264$$

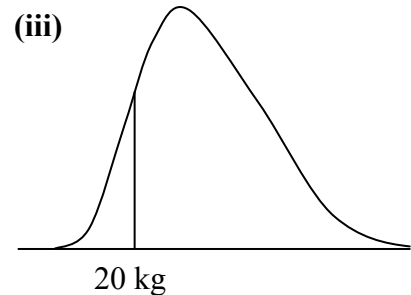
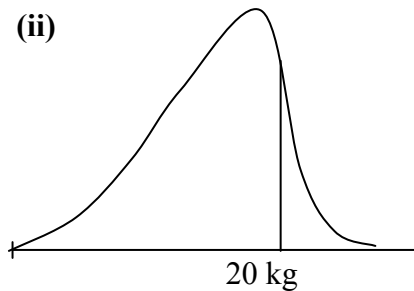
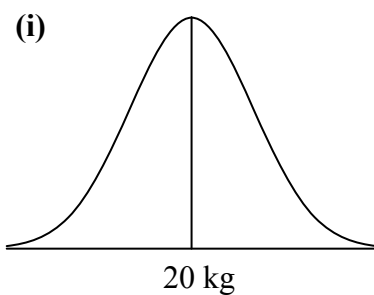
- (ii) An employee suggests that the probability of selecting a passenger whose flight was delayed and who was not satisfied with the overall service should be equal to the product of the two probabilities in (i) above. Do you agree with the employee?

Answer: No.

Reason:

If it was this would imply that the events were independent but this is not likely since a passenger who had his flight delayed is likely to be not satisfied with the service.

- (c) Which of the graphs below do you think is most likely to represent the distribution of the weights of passenger baggage?



Answer: Graph (ii)

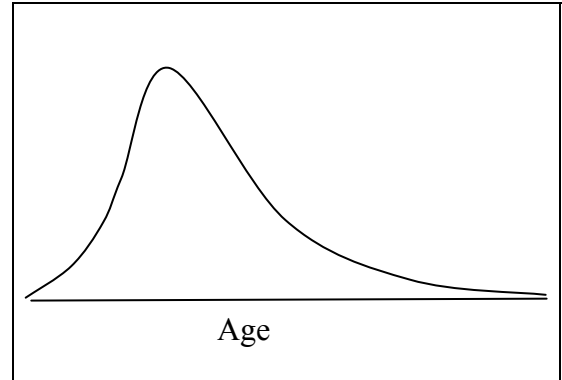
Reason:

A lot of the passengers are likely to have baggage with a weight of less than the maximum 20 kg.

- (d) (i) Draw a sketch of the possible distribution of the ages of the passengers based on the data in the survey.

- (ii) Explain your answer.

The median is less than the mean so the graph is skewed to the right.



- (e) (i) The company repeatedly asserts that 70% of their customers are satisfied with their overall service. Use an hypothesis test at the 5% level of significance to decide whether there is sufficient evidence to conclude that their claim is valid in May. State the null hypothesis and state your conclusion clearly.

Null Hypothesis: The satisfaction level is unchanged.  $p = 0.7$

The 95% margin of error for a sample of size 1000 is  $\frac{1}{\sqrt{1000}} = 0.0316$ .

The recorded satisfaction level for May is 0.664.

This is outside the range  $[0.7 - 0.0316, 0.7 + 0.0316] = [0.6684, 0.7316]$ .

Reject the null hypothesis.

There is evidence to conclude that the company claim is not valid in May.

**OR**

Null Hypothesis: The satisfaction level is unchanged.  $p = 0.7$

The 95% margin of error for a sample of size 1000 is  $\frac{1}{\sqrt{1000}} = 0.0316$ .

The 95% confidence interval for the population proportion is

$$0.664 - 0.0316 < p < 0.664 + 0.0316 = 0.6324 < p < 0.6956$$

$p = 0.7$  is outside this range.

Reject the null hypothesis.

There is evidence to conclude that the company claim is not valid in May.

- (ii) A manager of the airline says: "If we survey 2000 passengers from June on, we will half the margin of error in our surveys." Is the manager correct?

Answer: No.

Reason:

For a sample of size  $n$ , the margin of error is

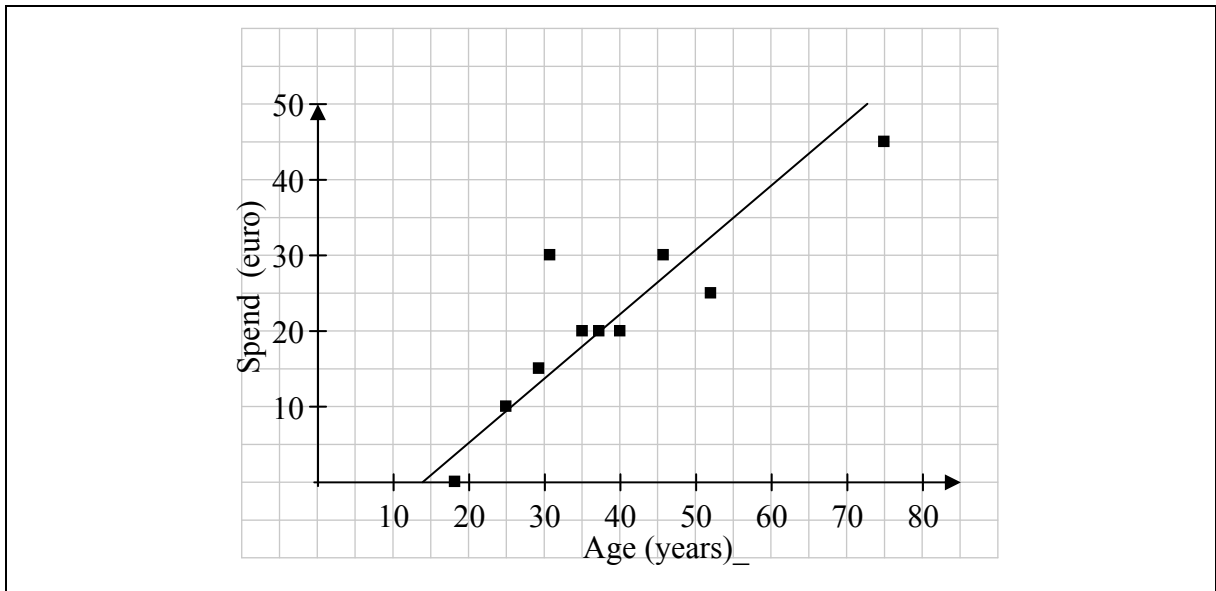
$$\frac{1}{\sqrt{n}} \cdot \frac{1}{2} \frac{1}{\sqrt{1000}} \neq \frac{1}{\sqrt{2000}}$$

or  $0.0158 \neq 0.022$

- (f) The responses of ten individual passengers to the questions on age and in-flight spend are given below.

Age (years)	46	29	37	18	25	75	52	35	40	31
In-flight spend (euro)	30	15	20	0	10	45	25	20	20	30

- (i) Draw a scatter plot of the data.



- (ii) Calculate the correlation coefficient between passenger age and in-flight spend.

0.88

- (iii) What can you conclude from the completed scatter plot and the correlation coefficient?

Older passengers tend to spend more.

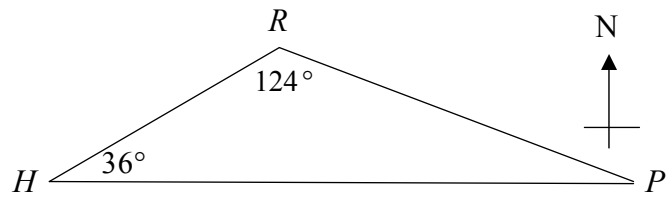
- (iv) Sketch the line of best fit in the completed scatter plot above.

**Question 8**

**(30 marks)**

Errors in this question mean that different valid approaches can result in different values for certain quantities. This will have caused difficulty for some candidates. See marking notes.

- (a) A port  $P$  is directly East of a port  $H$ . To sail from  $H$  to  $P$ , a ship first sails 80 km, in the direction shown in the diagram, to the point  $R$  before turning through an angle of  $124^\circ$  and sailing 110 km directly to  $P$ .



- (i) Find the distance from  $R$  to  $HP$ .

*Some possible solutions:*

$$\sin 36^\circ = \frac{d}{80} \Rightarrow d = 80 \sin 36^\circ = 47.02 \text{ km}$$

**OR**

$$\sin 20^\circ = \frac{d}{110} \Rightarrow d = 110 \sin 20^\circ = 37.62 \text{ km}$$

- (ii) Calculate  $|HP|$ .

*Some possible solutions:*

$$\begin{aligned} |HP|^2 &= 80^2 + 110^2 - 2(80)(110)\cos 124^\circ \\ &= 6400 + 12100 + 9841.79 = 28341.79 \\ |HP| &= 168.35 \text{ km} \end{aligned}$$

**OR**

$$\frac{|HP|}{\sin 124^\circ} = \frac{110}{\sin 36^\circ} \Rightarrow |HP| = 155.148 \text{ km}$$

**OR**

$$\frac{|HP|}{\sin 124^\circ} = \frac{80}{\sin 20^\circ} \Rightarrow |HP| = 193.915 \text{ km}$$

**OR**

$$|HP| = 80 \cos 36^\circ + 110 \cos 20^\circ = 64.72 + 103.366 = 168.087 \text{ km}$$

**OR**



$$\frac{1}{2}|HP|(47 \cdot 02) = \frac{1}{2}(80)(110)\sin 124^\circ$$

$$\Rightarrow |HP| = \frac{(80)(110)(0 \cdot 8290)}{47 \cdot 02} = 155 \cdot 15 \text{ km}$$

**OR**

$$|HP| = \frac{47 \cdot 02}{\tan 36^\circ} + \frac{47 \cdot 02}{\tan 20^\circ} = 64 \cdot 721 + 129 \cdot 186 = 193 \cdot 9 \text{ km}$$

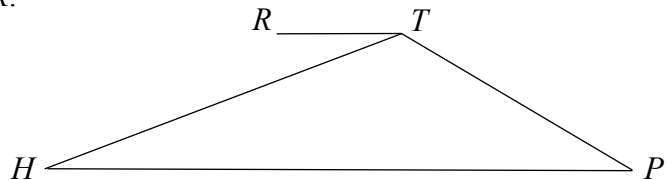
**OR**

$$|HP| = 47 \cdot 02 \tan 54^\circ + 47 \cdot 02 \tan 70^\circ = 193 \cdot 9 \text{ km}$$

(b) The point  $T$  is directly East of the point  $R$ .

$$|HT| = 110 \text{ km and } |TP| = 80 \text{ km.}$$

Find  $|RT|$ .



*Some possible solutions:*

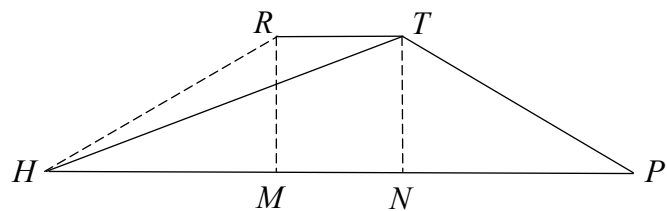
$$\cos 36^\circ = \frac{|HM|}{80}$$

$$\Rightarrow |HM| = 80 \cos 36^\circ$$

$$= 64 \cdot 72 \text{ km}$$

$$|NP| = 64 \cdot 72$$

$$|RT| = |MN| = 168 - 2(64 \cdot 72) = 38 \cdot 56 \text{ km}$$



**OR**

Taking  $\triangle HTP$

$$80^2 = 110^2 + 168 \cdot 35^2 - 2(110)(168 \cdot 35)\cos \angle THP$$

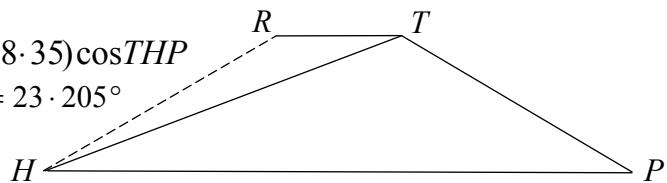
$$\Rightarrow \cos \angle THP = \cdot 9191 \Rightarrow \angle THP = 23 \cdot 205^\circ$$

$$\Rightarrow \angle RHT = 12 \cdot 795^\circ$$

Taking  $\triangle HRT$

$$|RT|^2 = 110^2 + 80^2 - 2(110)(80)\cos 12 \cdot 795^\circ$$

$$|RT|^2 = 1337 \cdot 0 \Rightarrow |RT| = 36 \cdot 56 \text{ km}$$



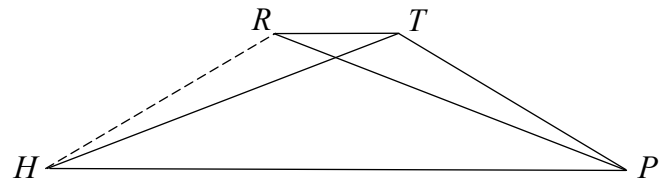
**OR**

$$|\angle RPH| = |\angle THP| = 20^\circ$$

$$\Rightarrow |\angle RHT| = 16^\circ$$

$$|RT|^2 = 110^2 + 80^2 - 2(110)(80)\cos 16^\circ$$

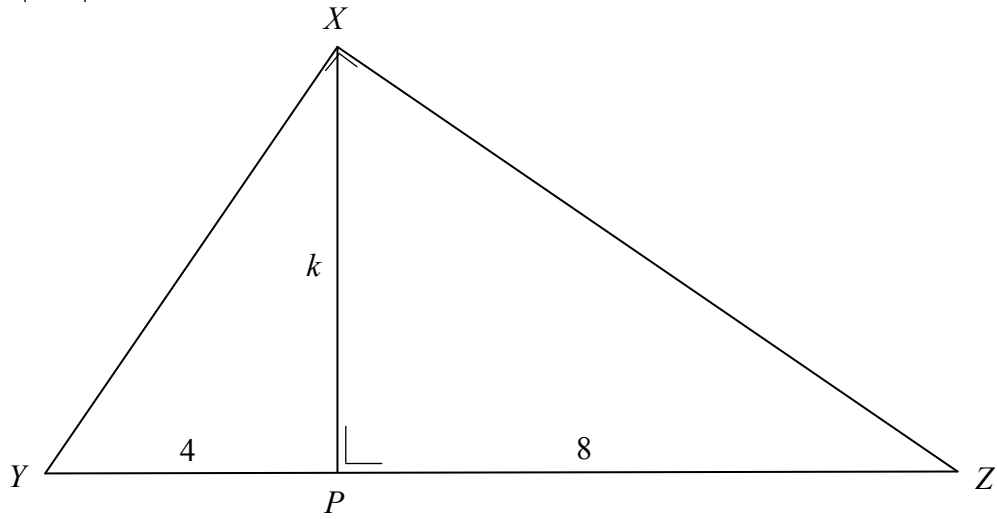
$$|RT|^2 = 1581.824 \Rightarrow |RT| = 39.77 \text{ km}$$



**Question 9**

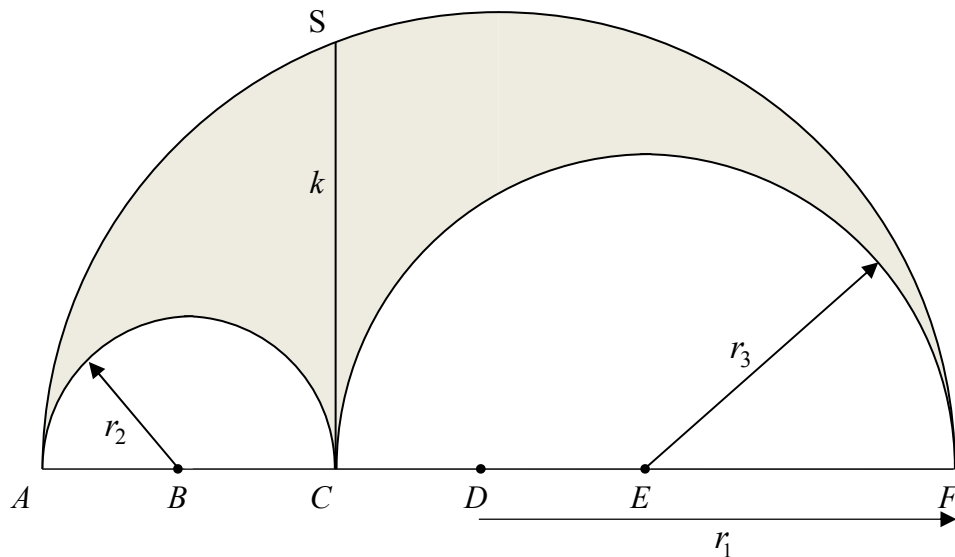
**(45 marks)**

- (a) The triangle  $XYZ$  is right-angled at  $X$  and  $XP$  is perpendicular to  $YZ$ .  
 $|YP|=4$ ,  $|PZ|=8$  and  $|PX|=k$ . Find the value of  $k$ .



$$\begin{aligned}
 |XY|^2 &= 4^2 + k^2 \Rightarrow k^2 = |XY|^2 - 16 \\
 |XZ|^2 &= 8^2 + k^2 \Rightarrow k^2 = |XZ|^2 - 64 \\
 2k^2 &= (|XY|^2 + |XZ|^2) - 80 = 144 - 80 = 64 \\
 \Rightarrow k &= \sqrt{32} = 4\sqrt{2}
 \end{aligned}$$

- (b) The shaded region in the diagram below is called an **arbelos**. It is a plane semicircular region of radius  $r_1$  from which semicircles of radius  $r_2$  and  $r_3$  are removed, as shown. In the diagram  $SC \perp AF$  and  $|SC|=k$ .



- (i) Show that, for fixed  $r_1$ , the perimeter of the arbelos is independent of the values of  $r_2$  and  $r_3$ .

$$\text{Perimeter} = \pi r_1 + (\pi r_2 + \pi r_3) = \pi(r_1 + r_2 + r_3) = \pi(r_1 + r_1) = 2\pi r_1$$

which is independent of  $r_2$  and  $r_3$

- (ii) If  $r_2 = 2$  and  $r_3 = 4$ , show that the area of the arbelos is the same as the area of the circle of diameter  $k$ .

$$\begin{aligned}\text{Area of arbelos} &= \frac{1}{2}\pi r_1^2 - \frac{1}{2}\pi(r_2^2 + r_3^2) \\ &= \frac{1}{2}\pi(6^2) - \frac{1}{2}\pi(2^2 + 4^2) \\ &= \frac{1}{2}\pi(36 - 20) \\ &= 8\pi\end{aligned}$$

$$k^2 + 4 = 36$$

$$k = \sqrt{32}$$

$$\text{Area of circle} = \pi\left(\frac{k}{2}\right)^2 = \pi\left(\frac{\sqrt{32}}{2}\right)^2 = \frac{\pi(\sqrt{32})^2}{4} = 8\pi$$

- (c) To investigate the area of an arbelos, a student fixed the value of  $r_1$  at 6 cm and completed the following table for different values of  $r_2$  and  $r_3$ .

(i) Complete the table.

$r_1$	$r_2$	$r_3$	Area of arbelos
6	1	5	$\frac{1}{2}\pi(6^2 - (1^2 + 5^2)) = 5\pi \text{ cm}^2$
6	2	4	$\frac{1}{2}\pi(6^2 - (2^2 + 4^2)) = 8\pi \text{ cm}^2$
6	3	3	$\frac{1}{2}\pi(6^2 - (3^2 + 3^2)) = 9\pi \text{ cm}^2$
6	4	2	$\frac{1}{2}\pi(6^2 - (4^2 + 1^2)) = 8\pi \text{ cm}^2$
6	5	1	$\frac{1}{2}\pi(6^2 - (5^2 + 1^2)) = 5\pi \text{ cm}^2$

- (ii) In general for  $r_1 = 6$  cm and  $r_2 = x$ ,  $0 < x < 6$ ,  $x \in \mathbb{R}$ , find an expression in  $x$  for the area of the arbelos.

$$\begin{aligned}
 \text{Area of arbelos} &= \frac{1}{2}\pi r_1^2 - \frac{1}{2}\pi(r_2^2 + r_3^2) \\
 &= \frac{1}{2}\pi(r_1^2 - (r_2^2 + r_3^2)) \\
 &= \frac{1}{2}\pi(36 - (x^2 + (6-x)^2)) \\
 &= \pi(6x - x^2) \text{ cm}^2
 \end{aligned}$$

- (iii) Hence, or otherwise, find the maximum area of an arbelos that can be formed in a semi circle of radius 6 cm.

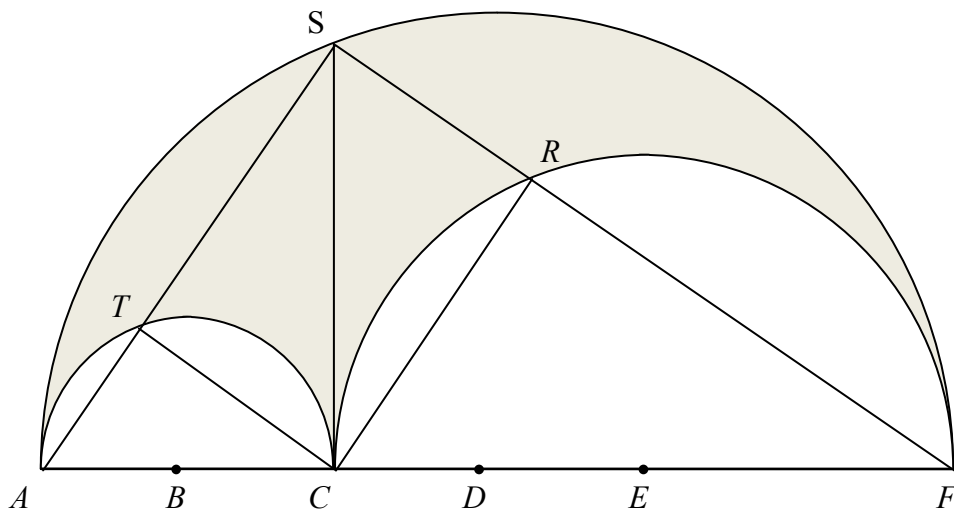
$$A = \pi(6x - x^2) \Rightarrow \frac{dA}{dx} = \pi(6 - 2x)$$

$$\pi(6 - 2x) = 0 \Rightarrow x = 3$$

$$\frac{dA}{dx} = \pi(6 - 2x) \Rightarrow \frac{d^2A}{dx^2} = -2\pi < 0 \Rightarrow \text{maximum}$$

Maximum area when  $x = 3$ , giving area =  $9\pi \text{ cm}^2$

- (d)  $AS$  and  $FS$  cut the two smaller semicircles at  $T$  and  $R$  respectively.  
Prove that  $RSTC$  is a rectangle.



$|\angle TSR| = 90^\circ$  ..... Angle in a semicircle

$|\angle CTA| = 90^\circ$  ..... Angle in a semicircle

Hence,  $|\angle STC| = 90^\circ$

$|\angle FRC| = 90^\circ$  ..... Angle in a semicircle

Hence,  $|\angle CRS| = 90^\circ$

Hence, the angles in  $RSTC$  are right angles and so  $RSTC$  is a rectangle.

## Marking Scheme – Paper 2, Section A and Section B

### Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5 mark scales	0, 5	0, 3, 5	0, 3, 4, 5	
10 mark scales		0, 5, 10	0, 3, 8, 10	0, 3, 7, 9, 10
15 mark scales			0, 5, 10, 15	0, 4, 7, 11, 15

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

### Marking scales – level descriptors

#### A-scales (two categories)

- incorrect response
- correct response

#### B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

#### C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

#### D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, *scale 10C\** indicates that 9 marks may be awarded.

## Summary of mark allocations and scales to be applied

### *Section A*

Question 1	
(a) (i)	5B
(a) (ii)	5B
(a) (iii)	5B
(b)	10D
Question 2	
(a) (i)	10C
(a) (ii)	5C
(b)	10C
Question 3	
(a)	10D
(b)	15D
Question 4	
(a)	10C
(b) (i)	10D
(b) (ii)	5C
Question 5	
(c)	5C
(d) (i)	10C*
(b) (ii)	5B
(e)	5B*
Question 6A	
(e)	10D
(f)	5B
(g)	10D
Question 6B	
(a)	10C
(b)	15C

### *Section B*

Question 7	
(a) (i)	5B
(a) (ii)	5B
(b) (i)	5B
(b) (ii)	5B
(c)	5B
(d) (i)	5B
(d) (ii)	5B
(e) (i)	10D
(e) (ii)	5B
(f) (i)	10C
(f) (ii)	5A
(f) (iii)	5B
(f) (iv)	5B
Question 8	
(a) (i)	10C*
(a) (ii)	10C*
(b)	10B*
Question 9	
(a)	10D
(b) (i)	5B
(b) (ii)	5B
(c) (i)	10B
(c) (ii)	5B*
(c) (iii)	5C*
(d)	5B



## Detailed marking notes

### Section A

#### Question 1

**(a)(i)** Scale 5B (0, 3, 5)

*Partial Credit:*

- Incomplete statement but with some merit
- Reference to outcomes
- Use of # symbol

**(a)(ii)** Scale 5B (0, 3, 5)

*Partial Credit:*

- Incomplete statement but with some merit
- Some reference to ‘common’ or ‘intersection’ of sets
- Venn diagram with E and F (or equivalent) shown but no reference to outcomes in common
- Reference to  $P(E \cup F)$

**(a)(iii)** Scale 5B (0, 3, 5)

*Partial Credit:*

- Incomplete statement but with some merit
- Reference to comparing/contrasting the outcomes of two events
- Reference to  $P(E \cap F)$  or  $P(E|F)$  or  $P(F|E)$

**(b)** Scale 10D (0, 3, 7, 9, 10)

*Low Partial Credit:*

- Venn Diagram with two or more entries correct
- Reference to  $P(E \cap F)$  only
- Reference to  $P(E)$  or  $P(F)$  only

*Mid Partial Credit:*

- Venn diagram with either  $P(E \cap F)$  or  $P(E)$  or  $P(F)$  or  $P(E|F)$  or  $P(F|E)$  calculated

*High Partial Credit:*

- $P(E \cap F)$  and  $P(E) \times P(F)$  found but correct conclusion not stated or implied
- Error in sample space but correct conclusion from candidate’s work

## Question 2

**(a)(i)** Scale 10C (0, 3, 8, 10)

*Low Partial Credit:*

- Any relevant step
- Formula written

*High Partial Credit:*

- Reference to 1.6
- Incorrect reading of tables

**(a)(ii)** Scale 5C (0, 3, 4, 5)

*Low Partial Credit:*

- Any relevant step other than in a(i)
- Diagram clearly indicating a 'new' area
- Reference to  $-1.6$
- $P(Z \leq -1.6) = 0.0548$  and stops

*High Partial Credit:*

- Probability of both situations calculated but fails to finish correctly
- Correct method with some error

**(b)** Scale 10C (0, 3, 8, 10)

*Low Partial Credit:*

- One correct
- Bell shape in one or more

*High Partial Credit:*

- Two correct

### Question 3

(a) Scale 10D (0, 3, 7, 9, 10)

*Low Partial Credit:*

- Any one correct
- Any line sketched correctly

*Mid Partial Credit:*

- Any two correct

*High Partial Credit:*

- Any four correct

(b) Scale 15D (0, 4, 7, 11, 15)

*Low Partial Credit:*

- Any reasonable step e.g. correct relevant formula
- Slope  $m$  or  $n$

*Mid Partial Credit:*

- Both slopes

*High Partial Credit:*

- $\tan \theta = \frac{1}{\sqrt{3}}$  and stops

#### Question 4

(a) Scale 10C (0, 3, 8, 10)

*Low Partial Credit:*

- Any one correct
- Any reasonable step

*High Partial Credit:*

- Any two correct
- Correct approach but error in work

(b)(i) Scale 10D (0, 3, 7, 9, 10)

*Low Partial Credit:*

- Some reference to ratio
- $c_1 \cap c_2$  and stops
- Any reasonable step

*Mid Partial Credit:*

- Some correct substitution into correct ratio formula
- Substitution resulting in a quadratic equation in one variable

*High Partial Credit:*

- Error in ratio formula but finishes
- Finds one of the ordinates only

(b)(ii) Scale 5C (0, 3, 4, 5)

*Low Partial Credit:*

- Slope of line of centres
- Any reasonable step

*High Partial Credit:*

- Uses point of contact with slope of diameter for equation
- $c_1 - c_2 = 0$  but not in linear format

## Question 5

(a) Scale 5C (0, 3, 4, 5)

*Low Partial Credit:*

- Relevant diagram
- One statement of area in trigonometric format
- Sine of a relevant angle in right angled triangle written in terms of sides
- Any reasonable step

*High Partial Credit:*

- Correct approach but one error in work

(b)(i) Scale 10C\* (0, 3, 8, 10)

*Low Partial Credit:*

- Relevant formula
- Any reasonable step

*High Partial Credit:*

- Error in substitution into formula but continues
- One value only
- Correct method but one error in work

(b)(ii) Scale 5B (0, 3, 5)

*Partial Credit:*

- One position only shown
- Triangle(s) sketched but  $Z$  not indicated

(c) Scale 5B\* (0, 3, 5)

*Partial Credit:*

- $|\angle ZXY|$  only
- Error in substitution into area formula
- Any reasonable step

## Question 6A

(a) Scale 10D (0, 3, 7, 9, 10)

*Low Partial Credit:*

- One partially correct statement
- One partially correct sketch
- Any reasonable step

*Mid Partial Credit:*

- One fully correct statement
- One fully correct sketch

*High Partial Credit:*

- Two correct statements
- Two correct sketches

(b) Scale 5B (0, 3, 5)

*Partial Credit:*

- Some relevant reference to side(s) of triangle
- Some relevant reference to angles of triangle
- An incomplete 'rough' sketch

(c) Scale 10D (0, 3, 7, 9, 10)

*Low Partial Credit:*

- Some correct elements of construction
- Some evidence of understanding of term orthocentre
- Any reasonable step

*Mid Partial Credit:*

- One correct altitude

*High Partial Credit:*

- Two altitudes but not intersecting

## Question 6B

(a) Scale 10C (0, 3, 8, 10)

*Low Partial Credit:*

- Any correct step e.g.,
  - Identifies two equal sides
  - Identifies two equal angles

*High Partial Credit:*

- Proof with correct steps but without justification of steps
- Congruent triangles established but fails to complete
- No conclusion stated or implied

(b) Scale 15C (0, 5, 10, 15)

*Low Partial Credit:*

- Any correct step e.g.,
  - Establishes parallel sides
  - Identifies two equal sides

*High Partial Credit:*

- Proof with one step not fully established
- No conclusion stated or implied

## Section B

### Question 7

**(a)(i)** Scale 5B (0, 3, 5)

*Partial Credit:*

- Reference to subgroups
- Reference to sampling
- Definition of random sample

**(a)(ii)** Scale 5B (0, 3, 5)

*Partial Credit:*

- Not clearly disjoint groups
- Incomplete number of groups (at least two)

**(b)(i)** Scale 5B (0, 3, 5)

*Partial Credit:*

- One correct probability
- Some evidence of relevant understanding

**Note:** Accept any other answer in the range  $[0.231, 0.252]$ , provided a suitable rationale is given

**(b)(ii)** Scale 5B (0, 3, 5)

*Partial Credit:*

- Correct answer without explanation
- Correct answer with incorrect or incomplete explanation

**(c)** Scale 5B (0, 3, 5)

*Partial Credit:*

- Correct answer without explanation
- Correct explanation but omits to nominate correct graph

**(d)(i)** Scale 5B (0, 3, 5)

*Partial Credit:*

- A partially correct bell shape curve

**(d)(ii)** Scale 5B (0, 3, 5)

*Partial Credit:*

- Mention of mean and median but interpretation not related to candidate's curve



**(e)(i)** Scale 10D (0, 3, 7, 9, 10)

*Low Partial Credit:*

- One relevant step e.g. null hypothesis stated only
- Some work towards margin of error

*Mid Partial Credit:*

- Substantive work with one or more critical omissions
- Margin of error and range found but fails to continue

*High partial Credit*

- Failure to state null hypothesis correctly
- Failure to contextualise answer (e.g. stops at 'Reject null hypothesis')

**Note:** Accept candidate work based on disregarding *don't knows*, yielding an observed satisfaction rating of 664/902 and a corresponding  $n = 902$ .

**(e)(ii)** Scale 5B (0, 3, 5)

*Partial Credit:*

- Correct answer but no explanation
- Partially correct explanation

**(f)(i)** Scale 10C (0, 3, 8, 10)

*Low Partial Credit:*

- Correct scale with at least two points plotted

*High Partial Credit:*

- Correct scales but not all points plotted (one or two omissions)
- All points plotted but scales incorrect

**(f)(ii)** Scale 5A (0, 5)

**(f)(iii)** Scale 5B (0, 3, 5)

*Partial Credit:*

- Partially correct answer e.g. not in context
- Positive or strong positive correlation and stops

**(f)(iv)** Scale 5B (0, 3, 5)

*Low Partial Credit:*

- Straight line but clearly not best fit

## Question 8

**(a)(i)** Scale 10C\* (0, 3, 8, 10)

*Low Partial Credit:*

- Identifies a relevant right angle
- $\sin 36^\circ = \frac{\text{opp}}{\text{hyp}}$  or equivalent

*High Partial Credit:*

- $\sin 36^\circ = \frac{d}{80}$  or equivalent (e.g.  $\sin 20^\circ = \frac{d}{110}$ )

**Note:**(i) Accept candidate answer from this section if and when used in later sections.

(ii) Units - apply penalty once only in question

(iii) Do not penalise candidates for rounding off answers in **(a)(i)** and **(a)(ii)**.

**(a)(ii)** Scale 10C\*(0, 3, 8, 10)

*Low Partial Credit:*

- Identifies Cosine Rule
- $\cos 36^\circ = \frac{\text{adj}}{\text{hyp}}$  or  $\cos 20^\circ = \frac{\text{adj}}{110}$  or  $\tan 36^\circ = \frac{d}{\text{adj}}$  or equivalent statements

*High Partial Credit:*

- $|HP|^2$  calculated and stops
- Substantially correct work with one error

**(b)** Scale 10B\*(0, 5, 10)

*Partial Credit:*

- Some relevant work

**Note:** Where there is no evidence of impact of error, mark according to candidate's work. There are several correct approaches. Track candidate's data throughout, accepting any valid approach to each part.

Where there is evidence of impact of the error, award full marks in the part in which the impact occurs and subsequent part(s). Ensure that all such scripts are forwarded for review.

**Criteria for identifying impact of error:**

- Attempts a part more than once and gets different values for the same length.  
For example:
  - In part **(a)(i)**, calculates  $d$  as  $80 \sin 36^\circ = 47.02$  **and as**  $110 \sin 20^\circ = 37.62$ .
  - Calculates  $|HP|$  using the cosine rule and the sine rule (or other method), getting different values.
- Calculates  $|HR|$  as  $110 \frac{\sin 20^\circ}{\sin 36^\circ} = 64$ , contradicting an identified value of 80 for  $|HR|$ .
- Correctly generates a value for any angle that conflicts with an already calculated or identified value for the same angle.
- Values correctly obtained lead to a value of cosine or sine that is outside the range  $[-1, 1]$ .
- Attempts to construct an accurate scaled diagram and encounters difficulty.
- Any explicit statement suggesting awareness of conflicting data.

If you encounter any other evidence of potential impact, not covered by the above, please notify your Advising Examiner immediately.

## Question 9

(a) Scale 10D (0, 3, 7, 9, 10)

*Low Partial Credit:*

- One correct statement in  $k^2$
- One correct ratio involving  $k$
- Some relevant use of Pythagoras
- Any reasonable step e.g. establishing similar triangles

*Mid Partial Credit:*

- Two correct statements in  $k^2$
- Two correct ratios involving  $k$

*High Partial Credit:*

- $k^2$  evaluated, but  $k$  not found

(b)(i) Scale 5B (0, 3, 5)

*Partial Credit:*

- One relevant perimeter
- $r_2 + r_3 = r_1$  written or implied
- Any reasonable step

(b)(ii) Scale 5B (0, 3, 5)

*Partial Credit:*

- Area of one semi circle
- Area of circle with diameter  $k$
- Area of arbelos only

(c)(i) Scale 5B (0, 5, 10)

*Partial Credit:*

- Two correct entries

(c)(ii) Scale 5B\* (0, 3, 5)

*Partial Credit:*

- Identifies  $r_3 = 6 - x$
- Area of either semicircle in terms of  $x$

(c)(iii) Scale 5C\* (0, 3, 4, 5)

*Low Partial Credit:*

- Some correct differentiation
- Incomplete quadratic graph sketched

*High Partial Credit:*

- $x = 3$  established as maximum but area not calculated
- Graphical result not interpreted

(d) Scale 5B (0, 3, 5)

*Partial Credit:*

- One relevant step

## Marcanna breise as ucht freagairt trí Ghaeilge

### (Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g.  $198 \text{ marc} \times 5\% = 9.9 \Rightarrow$  bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle  $[300 - \text{bunmharc}] \times 15\%$ , agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0

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